VOLUME LIV NUMBER 3

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THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

EDITED BY

GEORGE E. HALE
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OCTOBER 1921

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VOLUME LIV OCTOBER 1921 NUMBER 3

THE FLUORESCENCE OF MERCURY VAPOR

By J. S. VAN DER LINGEN and R. W. WOOD

ABSTRACT

The Fluorescent spectrum of mercury vapor cannot be excited in quiescent vapor, but only in vapor which is being distilled from the metal at a temperature of 150° or more. This proves that the *active molecules* are not neutral monatomic molecules but others present only during distillation, perhaps diatomic molecules. The spectrum consists of three lines—λ 2536, 2539, and 2346 A—and four structureless bands with maxima at 2346, 2540, 3300, and 4850 A. While the complete spectrum is excited by light from a zinc spark, single lines excite only part. *The relation of the spectrum to the exciting light* is of great interest. The broad bands with maxima at 3300 and 4850 may be excited, with varying relative intensity, by λ 1854–62, 2346, or 2536–40 A; whereas the band λ 2346–2100 A is excited only by λ 2000–2150 A, the position of the maximum depending on the particular wave-length used. All the effective exciting lines lie, of course, in the absorption spectrum of mercury vapor.

The fluorescence of mercury vapor was first observed by Hartley while he was carrying out experiments on the absorption spectra of the elements and their compounds.[1]

He found that the hot vapor, which was contained in an unexhausted quartz vessel, showed an absorption band which extended from 2526.8 A to 2571.67 A and that it fluoresced when it was excited by either a lead-cadmium or a tin-cadmium spark, the color of the fluorescent light being bluish green. He observed that the fluorescent light was most brilliant when the mercury was boiling briskly and that the fluorescence ceased when all the mercury had been converted into vapor above the boiling-point of the metal.

[1] *Proceedings of the Royal Society*, **76**, 428, 1905.

From these observations he concluded, (*a*) that the phenomenon was not due to oxidation, because of the expulsion of the air during the brisk boiling of the metal, and (*b*) that there must be an upper and a lower limit of temperature between which fluorescence occurs.

Subsequently Professor R. W. Wood proved that mercury vapor *in vacuo*, at room temperature, shows two absorption lines, viz., 2536.7 A and a fainter one at 2539.3 A, and that these fuse together when the temperature is increased, while a band develops asymmetrically toward the red end of the spectrum, the head of the band remaining fixed at 2536.7 A.[1] If air be introduced into the bulb, the head first grows symmetrically for about 8 angstrom units on either side of 2536 A, and then extends toward the red end only. It was thus due to the presence of air that Hartley found the absorption band extending to 2526.8 A. Wood also found that there are absorption lines at 2346.1, 2339, 2334.4, and 2331.2 A and that these lines shaded off toward the violet end of the spectrum. If the density of the vapor be increased, these lines fuse together and form a band. The development of this band is difficult to trace because it is blotted out by the absorption band which heads from the absorption line 1849 A to the visible spectrum. He found that the fluorescent spectrum of the vapor *in vacuo* is continuous and extends "roughly from the yellow down to wave-length 3000 with a very pronounced minimum at wave-length 3600," and that these bands show very little change, if any, when the vapor is excited by isolated lines of a spark spectrum. In addition to these fluorescent bands Wood found an emission line at 2536.7 A, and a fainter companion line 2539.4 A when the vapor is excited by an aluminum or cadmium spark, the emission being due to the last aluminum line and a group of cadmium lines which lie near the head of the 1849 absorption band. In the latter case, however, the emission line is much weaker than in the case of aluminum excitation. "This line appears strongest when the vapor density is such that the visible fluorescence does not quite appear. As the temperature is raised and the visible fluorescence grows stronger, the 2536 line fades away." Furthermore he showed that the fluores-

[1] *Philosophical Magazine* (6), **18**, 240, 1909.

cence disappears when the vapor is superheated, and also when air is introduced into the bulb.

Immediately after the appearance of Wood's results Steubing published his observations on the fluorescence of mercury vapor, and he showed that not only is the mercury line 2536 A emitted, but also the mercury line 2346 A, when the vapor is excited by an aluminum spark. He also found a fluted band, which extends from this latter line to about 2100 A. Furthermore he found that the ionization of the vapor was increased when it was excited by radiations lying within an absorption band of short wave-length. These results, he maintains, are in conformity with Stark's theory of absorption and fluorescence.[1]

In 1914 Phillips[2] examined the resonance phenomenon of this vapor, which was discovered by Wood, i.e., the emission of 2536 A when the vapor at room temperature is excited by 2536 A of a water-cooled quartz-mercury arc. In order to determine the percentage of secondary resonance radiation, he allowed the vapor to distil past a horizontal slit and discovered that the vapor emitted 2536 A as well as the fluorescent bands after it had passed out of the tract of the exciting light. He maintains that there are four fluorescent bands which from their position on his spectrum appear to be at the following positions approximately: (1) a diffused band at about 2700 A; (2) heads on 3132 A and ends at 3650 A; (3) at 4360 A; (4) a narrow band on the side of this region toward long wave-lengths. From what has been said it appears that mercury has absorption lines at 1849, 2346, and 2536 A and three absorption bands: (1) heads from 1849 A to the visible spectrum; (2) consists of the fused lines at 2346 A which develop into a band; (3) extends asymmetrically from 2536 A to the visible spectrum as the temperature is increased.

With regard to the work on the fluorescent bands and lines the results of the different observers do not agree.

OBJECT OF RESEARCH

The object of the following work was to find, as far as possible, the exact conditions under which mercury vapor fluoresces, and

[1] *Physikalische Zeitschrift*, **10**, 787, 1909.

[2] *Proceedings of the Royal Society*, **89**, 39, 1914.

also to determine the correlation between excitation in the various absorption bands and the emitted fluorescent lines and bands.

In the course of a somewhat extended study of the fluorescence of the vapor made by one of us, phenomena have from time to time been noticed which indicate that the fluorescence is not due to the monatomic mercury molecule. The evidence was thus summarized in a monograph on fluorescence not yet in print:

> A thin-walled bulb, highly exhausted and containing a drop of mercury was heated in an Heraeus electric furnace and illuminated by an aluminum spark placed close to the bulb. The temperature of the furnace was determined by a nitrogen-filled mercury thermometer, placed close to the quartz bulb. With the temperature rising very gradually a very feeble fluorescence was observed at 150° C. while at 170° C. it was very bright. The density of the mercury vapor is increased less than twofold by this temperature increase, whereas the intensity of the fluorescence was certainly tenfold, if not more. On cooling the furnace, fluorescence disappeared entirely as soon as the temperature had fallen three or four degrees, though the density was much greater than that at which good fluorescence had been obtained with a rising temperature.
>
> This makes it appear probable that something, diatomic molecules perhaps, commenced to come off the mercury at a temperature of 150°, the percentage of these fluorescing bodies increasing with rising temperature. As the temperature falls these bodies are no longer given off, and those already present disappear, probably by conversion into ordinary monatomic vapor.

The present investigation started out with a more careful study of this phenomenon. The experiment was repeated by using a similar quartz bulb which was placed over an asbestos chimney and gently heated by means of a Bunsen burner while a thermometer was placed over the bulb. On exciting the vapor by means of an aluminum spark placed near the bulb, it was found that the vapor exhibited faint fluorescence at 160° C., the intensity increasing rapidly with a rise of temperature.

On allowing it to cool, the fluorescence would cease at about 200°, and would only appear again on reheating up to 220° C. Very variable results were obtained by this method of heating, as was perhaps to be expected, but they verified completely the result of the earlier investigation, namely, that *the intensity of the fluorescence is a function of something other than the vapor-density.*

To secure a better control of temperature, an electric furnace was constructed of galvanized iron, lined with asbestos, the interior

being divided into two compartments by a vertical sheet of asbestos board. Two quartz bulbs joined by a short tube formed the mercury-vapor chamber, the tube passing through the asbestos partition. On either side of this partition an equal number of heating spirals were connected in series, and at the same time compensating spirals, having independent circuits, were fixed radially in each compartment. In this way the two connected bulbs could be kept at the same temperature, or a given difference of temperature between them could be established and held indefinitely. The exciting light entered the furnace as a parallel beam through a lateral hole, passing through the two bulbs and their connecting tube. In some cases the bulbs were illuminated one at a time by a beam entering the furnace at right angles to the axis of the bulb system. At constant temperature visible fluorescence could not be obtained; in other words, it was apparent that distillation must be going on in order to secure fluorescence. To settle this point definitely, the following very conclusive experiment was performed:

Two bulbs of very thin quartz, each about 1 cm in diameter, were joined by a short tube of quartz of about 1.5 mm bore. The two bulbs were mounted over a tall asbestos chimney, with a Bunsen burner at its base. The bulbs of two thermometers were placed immediately above the quartz bulbs. Operating under conditions avoiding air currents in the room, the two bulbs could be held at nearly the same temperature. One was always a little cooler than the other (about 5°) and in this one the liquid mercury of course collected.

When things had reached a steady state, say with one bulb at 200° and the other at 205°, both bulbs being illuminated by the light of an aluminum spark placed close to them, *no* fluorescence was observed. A blast of cold air, delivered through a small tube, was now directed against the hotter of the two bulbs, reducing its temperature enormously. The other bulb instantly flashed into a bright green fluorescence, which persisted until the last drop of mercury in it had disappeared, vanishing at the moment when distillation ceased.

The mercury was found now deposited as a dew on the cooled spot of the wall of the other bulb. The air current was now stopped and presently this bulb commenced to fluoresce as its temperature soon rose to its original value (205°) and the mercury began to distil back. No fluorescence was shown in either case in the bulb in which condensation was taking place. This was to be expected, as the pressure was lower in this part of the system.

However we interpret the results of this experiment, the fact is definitely established that only freshly formed mercury vapor is capable of exhibiting fluorescence.

This is not true of the resonance radiation described in previous papers by one of us, since mercury vapor in an exhausted tube at room temperature emits powerfully monochromatic light of wavelength 2536 A when illuminated by light of this same frequency obtained from a water-cooled quartz-mercury arc. Now it is known from the previous investigation just alluded to that the same radiation is also emitted by comparatively dense mercury vapor (say 4 mm pressure) when it is illuminated by the spark lines in the region 1860–1870 A.

It seems therefore desirable to ascertain whether this emission depended also upon distillation. This radiation appears first at much lower vapor-densities than any at which visible fluorescence does, which suggests that it very probably results from the stimulation of monatomic mercury molecules, and does not depend upon the presence of diatomic molecules or other unique entities.

The presence of this radiation can of course be detected only by the quartz spectrograph. The double bulb just described was used as before, the spectrograph being directed at the hotter bulb (250°). An exposure of 2 minutes was given, the bulb being illuminated with the light of the aluminum spark. A faint trace only of the 2536 Hg line appeared on the plate: no visible fluorescence was observed. This experiment was now repeated, and after an exposure of 2 minutes had been given, the spectrograph was screened off and the mercury brought over into the hotter bulb by an air blast. On removing the blast, the mercury commenced to distil back, and visible fluorescence occurred. Exposures were now made on the same plate of 2, 4, 6, and 8 seconds' duration.

On developing the plate it was found that the exposure of 2 seconds with the vapor distilling recorded the 2536 line with the same density as the 2 minutes' exposure of the stagnant vapor. This indicates that the 2536 emission line, excited by the short-wave-length spark lines, results in all probability from the stimulation of the entities which give rise to the visible fluorescence, but that a few of them are present even in the stagnant vapor, distillation increasing their number about sixty fold.

We will now consider the relation which exists between the wave-length of the exciting light, and the distribution of intensity among the various fluorescent bands and lines.

The absorption of mercury vapor is rather complex, and as one or two points in regard to it were still somewhat obscure, the investigation commenced with a study of this phenomenon. Combining previous work with our supplementary observations we arrive at the conclusion that the absorption spectrum is as represented in Figure 1*a*, viz.:

1. A strong absorption line at 2536 A, and a weaker one at 2539 A.

2. Four lines at 2346, 2339, 2334, 2331 A which fuse together with rise of temperature, forming a band which gradually pushes toward the side of short wave-lengths with increasing vapor-density.

3. There is in addition a strong absorption line at 1849 A corresponding to the powerful emission line at the same point.

4. Two strong and very asymmetrical absorption bands, one with its head in close proximity to the 2539 line and the other with its head near 1849 A. These bands creep out gradually, with increasing vapor-density toward the region of longer wave-lengths.

The complete fluorescence spectrum, which, for reasons which will be explained presently, is given by excitation with the zinc spark, is shown diagrammatically in Figure 1*b*.

It consists of: (1) a symmetrical structureless band, extending from the red down to wave-length 3700, with its maximum at 4850 A; (2) a similar broad band with a maximum at 3300 A, the two fusing together with long exposure; (3) an asymmetrical faint band coinciding with the absorption band just above 2539; and (4) a somewhat asymmetrical band extending from 2349 to 2100 A.

There are in addition the lines 2536 A and 2539 A, which, however, are not very strongly developed with the zinc spark.

It is of the utmost importance to find out how the absorption bands and lines are related to the fluorescent emissions.

For the examination of this point it is necessary to excite the vapor with monochromatic radiation or groups of such radiations lying in different regions of the absorption bands.

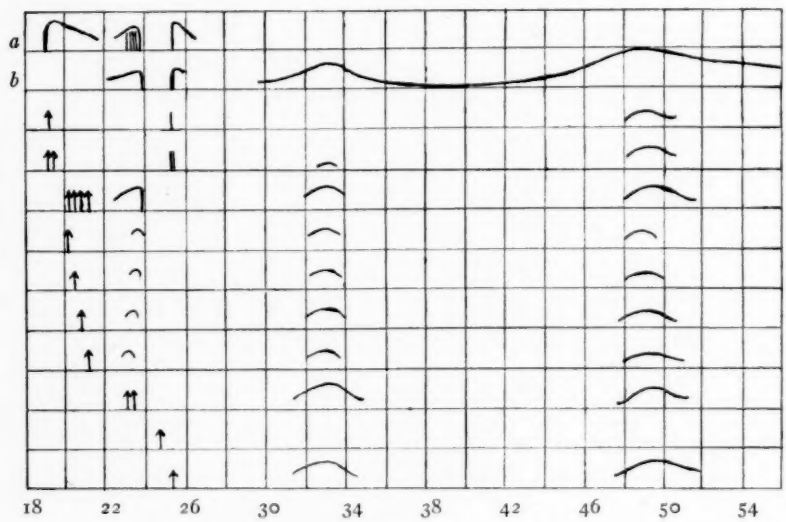

FIG. 1

a, absorption spectrum
b, fluorescent spectrum due to excitation by zinc spark
↑, exciting lines |, emission lines

Two methods have been employed for the isolation of such radiations, viz., prismatic decomposition of the light of the spark by means of two large quartz lenses with a quartz prism, and the method of focal isolation employed by Rubens and Wood for the isolation of very long heat waves.

In the former case two quartz lenses of 8 cm aperture and 50 cm focus were used in conjunction with two Cornu prisms, mounted on top of one another. In this case the bulb was placed immediately behind a tin screen, perforated with a slit and coated with

Willemite, on which the spark lines were focused. This screen rested on the funnel beneath which the Bunsen burner was placed, and the bulb was placed immediately behind the slit. By moving the quartz lens any desired spectrum line could be passed through the slit to the bulb.

In the second case the central portions of each lens were screened off by means of a black paper disk. The axes of the lenses and the spark were placed in line with a small, circular hole in a metal screen, coated with Willemite, behind which the bulb was placed. The desired groups of rays were then focused so as to pass through this hole on to the bulb. The object of the disk was of course to cut out the central beam of light.

This second method gives greater intensity of illumination, as the absorption of the prism is avoided, the full aperture of the lenses is utilized, and the greater transparency of the thin edges of the lenses for the very short wave-lengths comes into play. It suffers from the disadvantage that only a rather wide group of radiations can be isolated. Focal isolation with mercury fluorescence clearly shows the presence of the group of weak zinc lines between 1900 A and 1860 A. These lines are absorbed by the monochromator to such an extent that they have no effect on either the Willemite screen or the mercury vapor. To avoid the possible absorption of the fluorescent light by the vapor, the quartz spectrograph was directed toward the face of the bulb where the light entered in such a direction that little of the exciting light was reflected into the spectroscope. This condition was obtained by reducing the diaphragm of the spectroscope and observing the bulb through the instrument (eye placed at the focus) and arranging matters in such a way that none of the spark images seen reflected from the bulb walls fell within the aperture of the diaphragm.

EXCITATION IN THE 1849 ABSORPTION BAND

This absorption band begins at wave-length 1849 A and extends toward the visible spectrum with increase of vapor-density, reaching 2200 A at about 280° C., with a column of vapor 10 cm in length.

On separating the aluminum lines by means of the monochromator and allowing Al 1854 to excite the vapor, it was found that visible fluorescence first appeared at 210° C. and that the slightest increase in the temperature caused the fluorescent beam to recede to the face of the bulb.

On photographing the emission spectrum it was found that the excitation caused the emission of the 2536 line, and the continuous band in the green-violet (maximum at 4850). On increasing the density and photographing the emission spectrum again, it was found that the 2536 line was weaker, the green-violet band was more intense, and the ultra-violet band (maximum intensity at 3300 A) now appeared and grew stronger as 2536 A decreased in intensity. No trace of the 2539 line appeared, though this may have been due to insufficient exposure.

From this it appears that excitation at low vapor-density, with radiation at the head of the extreme ultra-violet band, causes a concentration of the emitted energy in the green-violet band; at higher vapor-density, however, the ultra-violet band appears and the 2536 line fades away. The same thing was observed if the excitation was caused by the Al lines in the region 1854–1862, though in this case there was a trace of the ultra-violet band, even at the low density, while at the higher density it was nearly as strong as the green-violet band. The development of the ultra-violet band is thus seen to be favored by excitation in a region a little removed from the head of the band.

It must be understood, however, that the wide ultra-violet absorption band may be a complex of several bands, each one associated with an emission band. If this were the case, we should then have the green-violet band associated with the component of the ultra-violet absorption band which falls just above the 1849 line, the ultra-violet emission band with a component of the absorption of somewhat longer wave-length, and if there were a third component of still longer wave-length, and we excited in this region, we should expect a third fluorescent band still farther down in the spectrum.

This is, in fact, the case, because on excitation by the group of zinc lines in the region 2000–2150 A we find a new fluorescent band,

not previously described, in the region 2349 to 2100. This band occupies approximately the position of Steubing's fluted band, of which we have never been able to find any trace. In addition, we find a trace of 2536 A and 2539 A and a faint band with its head at 2539 A which corresponds to the absorption band shown in Figure 1*a*.

The newly discovered fluorescent band at 2346 A behaves in a manner quite different from that of the bands in the green-violet and upper ultra-violet.

This band is excited in its entirety by the group of four zinc lines 2024, 2061, 2909, and 2138 A. If these are separated by the monochromator and the excitation is produced by each in turn, it is found that the region of maximum intensity in the new ultra-violet band moves to the violet end of the spectrum as the exciting line moves toward the visible end of the spectrum, i.e., excitation by the zinc line of shortest wave-length gives us only the portion of the emission band of longer wave-lengths, while excitation by the zinc line of longest wave-length gives only the short-wave-length portion of the band. Expressing this in other words, when we excite by the zinc lines in succession (increasing wave-length), the emission band moves down toward the excited region. If now we move the exciting wave-length still farther up the spectrum, utilizing a group of cobalt lines in the region of the group of absorption lines at 2346 A, we obtain the green-violet band and the next ultra-violet band (3300) of uniform intensity. The same is true if we excite with the monochromatic radiation of the 2536 Hg line obtained from the water-cooled mercury arc.

There is left now only the absorption band just above 2539 A. If we form an image of the two strong zinc lines (one of which lies to the right and the other to the left of the 2536 Hg line) on the surface of the quartz bulb, we find on heating the bulb that the zinc line of longer wave-length causes fluorescence. This line falls within the absorption band above referred to. The spectroscope shows that in this case we have also the green-violet and ultra-violet bands of uniform intensity.

On exciting the vapor by all the aluminum lines lying below 2360 a faint line appeared at 2346 A when the vapor was distilling

slowly. On increasing the vapor-density this line increases in intensity, but is soon blotted out by the extreme ultra-violet band which increases relatively faster in intensity than the line. In addition to the emission of 2346 A, first observed by Steubing, and the band in this region, 2536 A, 2539 A and the green-violet and ultra-violet bands are emitted.

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WAVE-LENGTHS AND PERIODIC CHANGES OF SPECTRAL TYPE IN THE VARIABLE STAR *l* CARINAE

By SEBASTIAN ALBRECHT

ABSTRACT

Cepheid variable l Carinae; periodic variations in wave-lengths and spectral type.—Since the wave-lengths of many lines in stellar spectra change progressively with spectral type it might be expected that in the case of a variable star periodic changes of wave-length might occur. A detailed study of *l* Carinae shows this to be true. The results of measurements of 178 lines between λλ 4236 A and 4495 A, made on 17 three-prism spectrograms obtained at Lick Observatory, are given in Table I. In general, those lines which vary in wave-length with stellar type vary progressively from light-maximum to light-minimum, shifting by about 0.05 to 0.2A. The spectrum may not correspond to a pure type at any phase, but by taking the average of the types indicated by the different lines for each phase, it is found that the mean type varies from F 8 near light-maximum to G 9 near light-minimum. From this change of type a change of color-index may be inferred which agrees with the changes observed or inferred for other Cepheids, showing that color is correlated not only to the hydrogen spectrum, as others have found, but also to the general spectrum. Discrepancies between these results, based on changes of wave-length, and those of Adams and Joy, based on changes of relative intensity and width of lines, are discussed.

Relation of the spectral types of Cepheids to their periods.—By tabulating the available data it is shown (1) that Cepheids are progressively of "later" type as the length of the period of light-variation increases; and (2) that the *range of variation of type* is about one type-interval and is independent of the period.

INTRODUCTION

In my first paper[1] on the determination of wave-lengths in stellar spectra of known types it was found that for many lines the wave-lengths remain substantially the same in all of the spectral types in which these lines are present. For numerous other lines, however, the wave-lengths change progressively[2] from type to type, so that for any particular line a smooth curve can be drawn through points plotted with $\Delta\lambda$ as ordinates and types as abscissae. In my early experience this was found to hold so rigidly that it was possible to estimate the stellar type—which had not previously been looked up—from the residuals of radial velocity, or of equivalent $\Delta\lambda$,

[1] *Lick Observatory Bulletin*, **4**, 90, 1906, and *Astrophysical Journal*, **24**, 333, 1906.

[2] Progressive in the sense of not having discontinuities, but with no restrictions as to direction of change.

for a few of the lines which are subject to the most pronounced changes in position. Subsequently this inversion of the process was employed for developing a method[1] whereby the spectral type can be quantitatively determined from the spectrograms of the star by a direct comparison of the measured wave-lengths with the curves representing the variations in the regular series of types.

At the time when the progressive changes in wave-length as a function of stellar type were discovered, it was pointed out[2] that similar changes of wave-length which would be progressive with the phase of light-variation might be found in the spectra of individual variable stars. Such periodic changes of wave-length, if found, would offer a quantitative method for the determination of actual changes in spectral type synchronously with the changes in light. The very limited amount of material then available, principally for the star η Aquilae, ". . . . showed very strong indications of just such variations in the positions of certain lines. . . . ."[3] At my suggestion Director Campbell kindly asked Dr. Curtis, then in charge of the Mills Observatory in Chile, to secure for this study series of spectrograms of the fourth-class variable stars l Carinae and κ Pavonis. The present paper discusses the measures of the spectrograms of l Carinae, $\alpha = 9^h42^m$; $\delta = -62°3'$ (1900); magnitudes 3.6–5.0; period 35.5 days, which were secured at that time.

PART I. WAVE-LENGTHS FROM 4236 A TO 4495 A, OBSERVED WITH THREE PRISMS

Seventeen three-prism spectrograms of l Carinae are available. Three of these (Nos. 299, 311, and 312) were taken with the short (16-inch) camera, and fourteen with the long ($21\frac{1}{4}$-inch) camera.[4] Hγ is central on the plates, and the Fe spark was used as source of light for the comparison spectra. All of the plates were measured

[1] *Astrophysical Journal*, **33**, 130, 1911.

[2] *Ibid.*, **24**, 335, 1906.

[3] *Lick Observatory Bulletin*, **4**, 131, 1907; and *Astrophysical Journal*, **25**, 334, 1907, footnote.

[4] *Publications of the Lick Observatory*, **9**, 56, 1907.

by the writer before leaving Mount Hamilton, except Nos. 1149, 1150, and 1201, which were measured by Dr. Olivier, and No. 312, which was measured by Dr. Palmer and the writer. Miss Hobe, Dr. Glancy, and Dr. Merrill assisted in the preliminary reductions. The methods of measurement and reduction employed are similar to those described in *Publications of the Lick Observatory*, **9**, and no large errors would be introduced by assuming the system to be the

TABLE I

λ for Radial Velocity	Number of Long-Camera Plates	λ for Radial Velocity	Number of Long-Camera Plates
4245.438	8	4379.383	7
50.296	13	83.720	13
50.955	13	87.007	14
54.502	9	91.147	14
74.955	9	95.255	14
88.143	14	99.917	14
92.304	13	4400.615	14
93.224	6	01.613	14
94.277	14	04.932	13
4306.921	14	07.853	13
13.043	13	08.579	8
15.178	11	17.863	13
18.870	14	20.686	14
20.975	14	27.444	14
25.195	14	28.715	12
28.104	14	41.875	13
34.075	14	42.529	14
37.227	10	47.915	13
39.713	4	50.654	13
52.018	13	59.317	14
52.949	14	66.749	14
71.363	14	68.663	13
4376.104	14	4476.211	13

same as for plates with numbers 500+ in that volume. However, in order to define the system more accurately for future use, the stellar wave-lengths employed in the reductions for radial velocity are given in Table I, together with the number of long-camera plates on which these lines were used. Half of these lines are of constant wave-length, both in *l* Carinae and in the corresponding types in the stellar series. The remaining lines vary in wave-length. Logically, only the stellar lines of constant wave-length should have been used in defining the system, and the reductions might

TABLE II

OBSERVED WAVE-LENGTHS FOR THE PRINCIPAL LINES WHICH WERE FOUND TO VARY IN *l* CARINAE

λ	312 2.1	1115 2.6	1120 3.6	1124 6.7	1147 11.6	1149 12.5	1040 17.1	299 17.6	1201 21.0	1209 21.9	1212 23.0	1220 24.0	1150 24.5	311 29.6	1079 30.1	1090 32.2	1098 33.2	Rowland λ in Sun	Rowland Inten. in Sun	Adams Inten. in Sun-Spots	Lockyer Enhanced El.	Lockyer Enhanced Spark	Lockyer Enhanced Arc
4258.4...	.427	.377	.387	.403	.428	.....	.....	.....	.....	.458	.489	.467	.....	.....	.460	.433	.377	.201	Zr 0	.....	Zr	2–3	1
																		.219	IN	0–1			
																		.477	Fe 2	4			
																		.639	ON				
																		.774	Fe 2				
4262.0...	.....	.120	.060	.096	.110	.....	.....	.....	.....	.043	.....	.043	.....	.010	.099	.032	.056	.891	2	.....	Zn	1	0
																		.086	1	0	Cr	5–6	0
																		.142	1				
4268.0...	.985	.009	.044	.047	.057	.....	.....	.....	.....	.033	.064	.043	.....	.....	.025	.005	.962	.900	1	.....	.....	.....	....
																		.985	Fe 3				
																		.138	Zr 0				
																		.266	IN				
4278.3...	.357	.371	340	.366	.351	.....	.....	.....	.....	.326	.401	.366	.....	.....	.363	.333	.289	.308	0	.....	.....	.....	....
																		.390	Fe-Ti 3	3–4			
4288.1...	.102	.127	.120	.163	.170	.149	.184	.179	.200	.162	.167	.147	.080	.039	.130	.122	.105	.038	Ti 2	.....	.....	.....	....
																		.149	Ni 1	.....	Ru	1	0
																		.310	Ti, Fe 1	2			
4313.0...	.001	.014	.029	.040	.022	.009	.....	.024	.017	.070	.992	.030	.062	.....	.047	.067	.077	.871	0	.....	.....	.....	....
																		.034	Ti 3	2	Ti	7	1–2
																		.193	IN				
4320.9...	.924	.979	.977	.967	.975	.927	.954	.968	.968	.934	.948	.957	.839	.000	.973	.983	.010	.661	0	1–2	.....	.....	....
																		.756	00				
																		.907	Sc 3	3			
																		.119	2	1	Ti	3	1
4331.8...	.853	.827	.829	.840	.815	.844	.822	.868	.824	.795	.811	.795	.802	.702	.801	.771	.779	.614	ON	.....	.....	.....	....
																		.811	Ni 2	2			
4334.0...	.056	.062	.072	.075	.078	.104	.092	.128	.086	.097	.071	.076	.081	.062	.065	.013	.035	.925	La 1N	.....	.....	.....	....
																		.063	00				
																		.174	00				
4344.5...	.561	.565	.563	.572	.582	.615	.624	.670	.630	.625	.662	.639	.692	.607	.581	.585	.566	.451	Ti– 2	n.c.	Ti	3	1
																		.670	Cr 4	6			
4352.9...	.954	.966	.958	.926	.943	.971	.921	.940	.969	.940	.959	.955	.020	.913	.945	.940	.919	.718	ON	.....	.....	.....	....
																		.908s	Fe 4	5–7			
																		.044	V 0				

4371.3...	.324	.334	.325	.344	.325	.356	.346	.392	.340	.347	.360	.332	.353	.289	.309	.286	.282	.144	Zr 1	1–2	Zr	3	1
																		.221	1				
																		.320	oo				
																		.442	Cr 2	3–4			
4390.1...	.....	.185	.144	.128	.141	.....	.137	.170	.....	*.165*	*.175*	.162	.....	.....	.102	.052	*.175*	.588	on d?				
																		.801	o	.....	.....	.....	....
																		.930	Mn, Ni? o				
																		.034	oo				
4391.9...	.998	.886	.902	.897	.912	.....	.916	*.848*	.....	.953	.949	.919	.....	.858	.908	.858	.857	.149	V 2	4			
																		.824	Co o	3	.....	.....	....
																		.924	Cr 1				
																		.034	Co o				
4395.2...	.214	.176	.196	.199	.189	.220	.242	.264	.261	.254	.261	.280	.217	.372	.242	.293	*.309*	.943	oo	o–1	Mg	1	o
																		.020	Zr o				
																		.201	Ti 3	2	Ti	9	5
																		.413	V, Zr 2	3			
4398.2...	.325	.297	.299	.279	.265	.....	.271	.297	.....	.277	.291	.278	.....	.339	.258	.298	.249	.178	1	.....	Yt	5–6	4
																		.338	oo				
																		.460	o				
4399.9...	.934	.906	.916	.910	.913	.887	.849	.846	.925	.915	.777	.882	.874	.....	.923	.907	.976	.776	Ni o	.....	.....	.....	....
4400.6...	.702	.685	.697	.659	.713	.638	.618	*.656*	.691	.726	.726	.713	.729	.....	.656	.644	.627	.935	Ti, Cr 3	2	Ti	7	3
																		.343	Zr on d?				
																		.555	Sc 3	3–4			
4403.4...	.420	.478	.496	.458	.445	.....	.433	.435	.....	.448	.445	.412	.....	.....	.441	.441	.....	.738	V 1	3			
																		.347	1	o–1	.....	.....	....
																		.532	Cr, Zr o	.....	Zr	1–2	o
4407.8...	.807	.844	.844	.844	.846	.852	.871	.....	.797	.860	.802	.806	.854	.918	.834	.794	.....	.810	V 2	8	.....	.....	....
																		.871	s Fe 4				
4412.2...	.169	.206	.216	.195	.224	.258	.286	.323	.230	.219	.241	.247	.238	.232	.182	.147	.126	.092	1	o	.....	.....	....
																		.297	V oo				
4416.9...	.947	.978	.958	.951	.988	.767	.833	.883	.802	.805	.807	.818	.771	*.927*	.850	.951	.967	.415	Cr o	1:0	Mo	2	o
																		.636	V o	1:1	.....	.....	....
																		.811	oon				
																		.985	2	1			
4417.8...	.891	.857	.854	.872	.847	.823	.819	.846	.789	.835	.862	.853	.919	.787	.866	.928	.938	.450	Ti o	1–2	.....	.....	....
																		.577	Co oo				
																		.740	oo				
																		.884	Ti– 3	n.c.	Ti	6	2
4422.0...	.065	.139	.109	.094	.030	*.854*	*.983*	.....	*.912*	*.777*	.873	.844	.810	.....	.921	.036	.....	.733	V o	1:1	.....	.....	....
																		.928	Ti oo	o	Ti	3	2
																		.104	1	oo–o			
																		.226	o				
4425.6...	.733	.652	.682	.654	.651	.....	*.676*	.881	.....	.807	.855	.799	.....	.....	*.690*	.657	.605	.608	s Ca 4	5–6	.....	.....	....
																		.827	oo				
																		.931	oon				
																		.121	ooo				
																		.201	Ti on d?	2			

TABLE II—*Continued*

λ	312 2.1	1115 2.6	1120 3.6	1124 6.7	1147 11.6	1149 12.5	1040 17.1	299 17.6	1201 21.0	1209 21.9	1212 23.0	1220 24.0	1150 24.5	311 29.6	1079 30.1	1090 32.2	1098 33.2	Rowland λ in Sun	Rowland Inten. in Sun	Adams Inten. in Sun-Spots	Lockyer Enhanced El.	Lockyer Enhanced Spark	Lockyer Enhanced Arc
4427.4	.465	.469	.453	.440	.437	.454	.459	.487	.488	.426	.481	.454	.518	.572	.505	.465	.484	.266	Ti 2	3	.....	.....	....
																		.482	Fe 5	7			
4428.7	.514	.....	.664	.701	.621	.648	.660	.727	.629	.646	.674	.623	.661	.583	.649	.603	.....	.711	V-Cr 1 d?	0:2	.....	.....	....
4429.3	.....	.387	.425	.377	.400	.....	.375	.....	.....	.393	.391	.384	.....	.334	.403	.475	.328	.366	∞	.....	.....	.....	....
																		.456	∞				
4435.2	.291	.194	.232	.237	.243	.....	.258	.....	.....	.283	.276	.....	.....	.284	.249	.207	.177	.1298	Ca 5	7	.....	.....	....
																		.321	Fe 2	3			
4441.8	.890	.834	.904	.844	.864	.850	.830	.849	.....	.825	.861	.864	.865	.....	.914	.875	.843	.881	V–3N d?	5	.....	.....	....
4444.7	.720	.....	.706	.....	.676	.....	.....	.....	.....	.667	.689	.620	.....	.659	.....	.....	.....	.728	Fe, Ti 2	1–2	.....	.....	....
4464.7	.764	.682	.727	.745	.744	.768	.785	.826	.804	.769	.792	.765	.779	.711	.736	.711	.705	.617	Ti? 2	n.c.	Ti	3	1
																		.844	Mn 2	3–4			
																		.938	Fe 1				
4466.7	.801	.705	.802	.758	.788	.779	.788	.797	.783	.771	.786	.756	.805	.790	.752	.749	.745	.548	Ni 0	.....	.....	.....	....
																		.727	Fe 5	7	K	1	0
4468.6	.693	.604	.642	.638	.648	.654	.690	.644	.669	.682	.741	.690	.704	.764	.702	.761	.....	.663	Ti– 5	4	Ti	9	4
4469.4	.467	.461	.514	.502	.513	.....	.545	.559	.....	.581	.534	.499	.....	.....	.480	.447	.....	.316	Ti 1	0–1	.....	.....	....
																		.441	∞∞∞				
																		.545	Fe 4	4			
																		.731	Co 0 d?				
																		.873	V ∞	0			
4472.9	.035	.010	.027	.010	.966	.....	.966	.986	.....	.927	.953	.920	.....	.969	.950	.953	.982	.884	Fe 1	2	.....	.....	....
																		.967	Mn 0				
																		.095	Ni? 0	∞			

NOTES

λ 4258.4 Broad and fuzzy. Curve weak for types F to K.

4262.0 Wave-length seems smaller near minimum, in conformity with sun-spot data.

4268.0 Increase published wave-lengths for types by +.091 A.

4278.39 Narrow in sun-spots.

4288.1 Blend according to intensities in sun, .134.

4320.9 Quite broad. Blend of .907 and .119, according to intensities in sun, .992.

4331.81 Very narrow line in sun-spots.

4344.5 Blend according to intensities in sun, .597.

4352.9 Stronger at minimum. Blend of .9 and .0 is widened to red in sun-spots.

4371.36 Broad blend. The increase in wave-length toward light minimum is to be expected from sun-spot data. This effect seems not to be shown in my measures in types.

4390.1 Curve of variation systematically higher than in types, as though the weaker components had been neglected in the measures.

4391.9 Weak on violet edge in sun-spots. No measures in types earlier than G is also significant.

4395.2 Possibly the enhanced Ti component is relatively much stronger at maximum than in types? Blend according to intensities in sun, .286.

4407.8 Only slight variation to be expected. Blend according to intensities in sun .851.

λ 4412.2 .415 is given as doublet in sun-spots.

4416.9 V given as doublet in sun-spots. The relative intensities of .636 and .985 change in *l* Carinae.

4417.8 Are the strengthened companions to the violet included in blend near minimum? Or, personality in settings on close pairs (toward minimum) only?

4422.0 Near minimum broad and fuzzy. These components exhibit well the changes of type with change of light phase.

4425.6 Broad, double, rather unsatisfactory; measured single at times; fuzzy field on side toward red. Ca is winged in sun-spots.

4427.4 Partly discordant. Fe winged in sun-spots. Blend according to intensities in sun, .420.

4428.7 Doublet in sun-spots.

4429.3 In G and K the range of variation is small.

4435.2 Ca is winged in sun-spots. Solar blend, .184.

4441.8 Very broad in sun-spots. Small range and weakly determined in types.

4464.7 Wave-length is systematically smaller in *l* Carinae. Is enhanced Ti relatively stronger in *l* Carinae than in types? Solar blend, .772.

4466.7 Fe "widened, hazy" in sun-spots.

4472.9 Adams, "widened to red. Rowland inten. too low: 2 better for total."

have been repeated, a posteriori, for this purpose. However, a detailed examination of the lines of variable wave-length shows that the increases and decreases of wave-length for the various phases of the star pretty closely balance each other, so that no appreciable gain would result in this case from a re-reduction. The details for the wave-lengths of the Fe comparison lines, the dispersion formula, and the correcting curve used with the formula, being the same for my more extensive measures of Southern Mills spectrograms with numbers 500+, more appropriately belong with those results and need not be duplicated here. Other data in regard to the spectrograms are available, if desired by anyone.

The observed wave-lengths (on the Rowland system, defined), obtained from the individual spectrograms, correct relatively to each other within errors of measurement and reduction, are given in Tables II and III, columns 2 to 18. With the exception of the double measures for Plate 312, each tabulated value was derived from a single complete measure, i.e., from two settings of the micrometer thread in the direct and two in the reversed positions of the plate. The arrangement of the spectrograms in a sequence according to phase of velocity and of light variation of the star, as well as the additional columns, are for purposes to be discussed in Part II. The figures directly below the plate numbers[1] in the headings give the phase-interval to the nearest tenth of a day, after maximum of light.[2] Italics signify that the line is especially poor in appearance. The means in the "remarks" column and footnotes refer to the long-camera plates only, except when it is apparent from the subscript that short-camera plates have also been included. The wave-lengths for the lines in stellar types are taken in part from *Boletin No. 1* of the Córdoba Observatory and in part from unpublished data. As these contain small systematic corrections which may be somewhat altered in a definitive discussion to be made later, they should not be employed at present to derive systematic corrections to the wave-lengths in *l* Carinae.

[1] The Roman numerals, which refer to the plate-holders, have been omitted in the plate numbers.

[2] No recent or good early light-curve is available.

TABLE III

SUPPLEMENTARY LIST OF WAVE-LENGTHS OBSERVED IN THE SPECTRUM OF *l* CARINAE

λ in Types	312 2.1	1115 2.6	1120 3.6	1124 6.7	1147 11.6	1149 12.5	1040 17.1	299 17.6	1201 21.0	1209 21.9	1212 23.0	1220 24.0	1150 24.5	311 29.6	1079 30.1	1090 32.2	1098 33.2
4236.079	.061																
4239.970		.969	.964	.968	.964						.884	.944					.909
4241.2								.258									
4242.6	*.601*							*.512*									
4245.438	.360	.417	.409		.440		.471	.366		.379	.393	.382				.370	
4246.163	.100	.154	.160	.141	.148		.142			.132	.152	.139			.175	.147	.103
4246.9	.978		.934														
4247.583	.648		.607		.624					.559		.637					
4250.2	.289	.286	.299	.293	.280	.314			.330	.297	.271	.334	.241		.331	.273	.248
4250.9	.962	.930	.952	.939	.986	.986			.986	.956	.958	.990	.969		.985	.963	.972
4254.5	.491	.471	.508	.477	.461					.498	.533	.533		.530	.594	.557	
4256.3	.338																
4266.0		.126	.097	.061	.063		.065			.053	.052	.053			.081	.048	
4267.0	*.007*	*.030*	.064	.065	.006					.026	.040	.064			.051	.022	.992
4268.8			.889	.907	.896					.924	.857	.897			.883		
4271.322	.139															.122	
4271.9	.885																
4273.5			.541														
4274.9	.885	.934	.910	.956	.931			.832		*.934*	.978	.971			.992	.975	
4275.7	.674	.752	.749	.760	.735					*.875*	.744	.770			.734	.761	
4276.84		.908	*.808*	.864	.859					.847	.792	*.853*					
4277.62	.516	.633	.637	.644	.606		.557			.586	.568	.576			.544		
4279.04					.075						.037						
4282.580	.449	.615	.536	.594	.583												
4283.160		.160	.169	.189	.135									.142		.200	
4284.5	.510				*.511*												
4285.617		.599	.595	.632	.616												
4287.0	.042																
4287.596					.598												
4290.2	.238																
4292.307	.304	.377	.324	.364	.335	.348			.352	.395	.372	.354	.385		.355	.350	.335
4293.2	.185				.295					.158	.200				.170	.143	.214
4294.277	.198	.234	.226	.241	.248	.227	.238	.183	.220	.162	.236	.213	.110	.154	.271	.264	*.294*
4295.0			.053		.098					.120	.082	.085		.120	.079		
4296.02	.088		.102		.040	.091		.081	.053	.052	.048	.053	.095	.052	.091		
4296.8		.842	.813								.951	.897			.869	.823	
4298.1	.078	.149	.086					.097		.088	.102	.070			.102	.103	
4304.6	.666														.630	.702	
4306.924	.918	.951	.907	.955	.920	.941	.921		.944	.942	.904	.941	.984		.898	.867	.885
4308.041	.005	.058	.995													.085	

4309.7	.....	.725	.765	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
4314.3	.317	.330	.264	.....	.312	.....	.....	.....	.....	.....	.378	*.324*	.345	.....	.357	.....	.341	.....
4315.1	.156	.214	.198	*.238*	.202	.....	*.244*	.....	.....	.....	.198	.254	.231	.....	.160	*.222*	.218	*.273*
4316.2	.282	.....	.....	.....	.....	.....	.....	.....	.....	.....	*.222*	.....	.....	.....	.....	.....	.....	.....
4317.1	.....	.010	.....	.....	.....	.....	.....	.....	.....	.....	.....	.030	.....	.....	.....	.....	.....	.....
	.150	.....	.....	.....	.255	.....	.....	*.235*	.....	.....	.157	.....	.....	.....	.163	.....	.....	.....
4317.4	.....	.....	.....	.....	.494	.....	.....	.....	.....	.....	.....	.467	.....	.....	.....	.....	.....	.....
4318.8	.954	.957	.921	.938	.888	.916	.921	.959	.901	.895	.908	.905	.878	.866	.855	.868	.872	
4319.7	.....	.....	.....	.....	.718	.....	.759	.728	.....	.689	.721	.....	.....	.877	.....	.....	.....	.....
4321.9	.....	.017	.004	.980	.928	.907	.....	.....	.....	.922	.933	.915	.....	.....	.926	.922	.....	
4322.6	.....	.727	.654	.669	.680	.....	.660	.....	.....	.686	.701	.671	.....	.....	.....	.639	.....	
4323.5	.....	.500	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
4325.1	.173	.138	.143	.142	.175	.175	.152	.....	*.204*	*.168*	.171	.198	.166	.224	.138	.166	.212	
4325.9	.905	.996	.992	.966	.969	.....	*.028*	.....	.020	*.093*	.046	.019	.....	.....	.967	.985	.025	
4327.16	.093	.127	.108	.099	.151	.....	.139	*.141*	.....	.126	.125	.122	.....	.112	.122	.072	.171	
4328.104	.151	.113	.087	.110	.097	.170	.086	.094	.113	.110	.092	.108	.101	.151	.080	.124	.091	
4329.1	.171	.....	.....	.....	.....	.....	.....	.334	.....	.227	.200	.180	.....	.....	.217	.....	.....	
4330.7	.718	.703	.717	.709	.697	.....	.....	.....	.....	.629	.629	.659	.....	.780	.664	.752	.....	
4332.9	.884	.942	.922	.896	.895	.....	.886	.949	.....	.874	.917	.906	.....	.882	.887	.914	.....	
4335.00	.....	.....	.....	.....	.....	.....	.950	.....	.....	.942	.993	.....	.....	.029	.....	.....	.....	.....
4336.4	.436	.....	.....	.....	.....	.....	.....	.....	.....	.....	.387	.....	.....	.....	.....	.....	.....	.....
4337.227	.312	.274	.289	*.259*	.212	.....	.204	.291	.....	.....	*.243*	*.256*	.....	.....	.207	.232	.219	
4338.0	.126	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
4339.713	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.717	.....	.....	.630	.652	.712	.651	
4340.6	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	*.624*	.....	.....	.....	.....	.....	.....	.....
4341.0	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.024	.....	.....	.....	.....
4341.5	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.456	.....	.....	.....	.....	.....	.....	.....
4342.3	.....	.341	.....	.356	*.350*	.....	.330	.362	.....	.337	.337	.341	.....	.....	.376	.302	.....	
4346.0	.....	.....	.....	.....	.....	.....	.....	.081	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
4346.8	.872	.....	.....	.....	.762	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
4347.46	.....	.....	.....	.....	.465	.....	.....	.....	.....	.....	.....	.....	.....	.461	.....	.....	.....	.....
4348.0	*.949*	.....	.....	.....	.023	.....	.....	.....	.....	.....	.....	.....	.....	.164	.....	.....	.....	.....
4349.14	.124	.120	.192	.121	.170	.....	.126	.132	.....	.129	.141	.134	.....	.....	.116	.088	.....	
4349.9	.982	.998	.958	.980	.980	.....	.978	.080	.....	.918	.926	.912	.....	.....	.967	.966	.....	
4351.1	.....	*.163*	.228	.180	.162	.....	.....	.....	.....	.....	.207	.....	.....	.....	.177	.145	.....	
4352.0	.950	.006	.946	.022	.014	.024	.....	.980	.041	.109	.038	.017	.031	.....	.992	.021	*.004*	
4354.7	.708	.745	.734	.710	.710	.....	.743	.898	.....	.....	.....	.....	.....	.....	.862	.790	.....	
4355.29	.....	.....	.311	.240	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
4356.12	.....	.....	.119	.136	.....	.....	.146	.....	.....	.....	.....	.....	.....	.....	.....	.105	.....	
4358.3	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.370	.....	
4358.8	.726	.....	.804	.....	*.766*	.....	.....	.791	.....	.678	.748	.656	.....	.....	.638	.872	.....	
4359.8	.830	.833	.837	.791	.797	.....	.822	.838	.....	.798	.767	.774	.....	.700	.771	.822	.779	
4360.9	.....	.....	.972	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.853	.....	.....	.....	.....
4362.2	.157	.....	.176	.....	.....	.....	.240	.187	.....	.087	.111	.....	.....	.....	.....	.....	.....	.....
4363.4	.423	.....	.412	.....	.....	.....	.....	.402	.....	.....	.430	.....	.....	.....	.....	.....	.....	.....
4364.8	.....	.....	.820	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
4366.3	*.349*	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....

TABLE III—*Continued*

λ in Types	312 2.1	1115 2.6	1120 3.6	1124 6.7	1147 11.6	1149 12.5	1040 17.1	299 17.6	1201 21.0	1209 21.9	1212 23.0	1220 24.0	1150 24.5	311 29.6	1079 30.1	1090 32.2	1098 33.2
4367.9	.033	.990	.928	*.970*	*.952*	.....	.....	.....	.....	.....	.983	.....	.....	.....	.....	*.922*	.922
4369.9	.900	.812	.807	.....	.....	.....	.888	.938	.....	.856	.858	.850	.....	.....	.834	.815	.....
4373.7	.760	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
4376.1	.104	.108	.096	.098	.111	.105	.131	.066	.096	.108	.117	.117	.134	.094	.115	.121	.093
4379.3	*.420*	.....	.....	.....	.382	.....	.388	.385	.....	.362	.373	.402	.....	.494	.382	.383	.....
4382.1	.....	.....	.....	.....	.169	.....	.219	.....	.....	.....	.....	.....	.....	.209	.....	.....	.....
4383.721	.660	.702	.708	.717	.724	.701	.....	.....	.764	.....	*.689*	*.634*	*.793*	.....	.745	.755	.776
4385.0	.....	.....	.....	.....	*.040*	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
4385.3	.....	.....	.....	.....	.....	.....	.....	.338	.....	.....	.....	.....	.....	.....	.....	.....	.....
4385.6	.....	.....	.....	.....	*.646*	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
4387.0	.962	.982	.957	.963	.962	.957	.962	.041	.975	.909	.953	.962	.917	.938	.937	.960	.979
4388.0	.....	.....	.....	.....	.035	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
4389.4	.450	.400	.406	.407	.403	.....	.415	.434	.....	.414	.454	.406	.....	.....	.390	.406	.336
4391.14	.116	.129	.100	.110	.148	.077	.156	.105	.118	.169	*.099*	.113	.115	.986	.138	.115	.091
4393.4	.....	.....	.....	.....	.418	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
4394.2	.197	.200	.214	.200	.217	.....	.191	.287	.....	.190	.188	.185	.....	.200	.166	.218	.203
4396.01	.....	.050	.053	.043	.053	.....	.049	.....	.....	.998	.087	.032	.....	.984	.985	.971	.....
4401.613	.648	.634	.622	.628	.632	.642	.657	*.634*	.616	.614	.598	.606	.626	.635	.601	.565	.562
4404.932	.942	.922	.950	.958	.958	.917	.....	.907	.911	.900	.901	.923	*.811*	.023	.947	.928	.991
4406.81	.....	.....	.....	.....	.767	.....	.769	.833	.....	.781	.817	.782	.....	.....	.769	.....	.....
4408.579	.648	.601	.620	.....	.601	.....	.....	.....	.....	.654	.646	.614	.....	*.679*	*.629*	.610	.....
4409.5	.....	.558	.548	.....	.533	.....	.....	.....	.....	.575	.525	.526	.....	.505	.519	.525	.....
4411.0	.057	.....	.099	.....	.....	.....	.....	.028	.....	.983	.994	.962	.....	.....	.995	.....	.....
4413.7	.874	.812	.834	.....	.851	.....	.....	.....	.....	.807	.....	.814	.....	.780	.....	.781	.....
4415.4	.443	.....	.456	.....	.....	.....	.....	.....	.....	.375	.374	.371	.....	.....	.361	.410	.452
4418.5	.....	.....	.....	.....	.510	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
4420.694	.789	.758	.730	.726	.695	.683	.710	.718	.677	.680	.676	.671	.634	.....	.670	.718	.682
4422.7	.....	.....	.797	.....	.778	.....	.....	.....	.....	.....	.....	.....	.....	.....	.782	.817	.....
4423.0	.....	.....	.....	.....	.....	.....	.....	.045	.....	.928	.905	.....	.....	.....	.....	.....	.....
4424.3	.527	.....	.....	.....	.....	.....	.....	*.342*	.....	.361	.360	.327	.....	.257	*.417*	.....	.....
4430.1	.....	.....	.....	.....	.162	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
4430.7	.....	.....	.....	.....	.741	.....	.....	.714	.....	.....	.....	.....	.....	.....	.....	.....	.....
4431.5	.....	.....	.....	.....	.525	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
4433.3	*.393*	.....	.....	.....	.365	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
4434.0	.043	.....	.....	.....	.020	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
4435.4	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.440	.....	.....	.....	.411	.....
4435.7	.....	.719	.753	.790	.750	.....	.805	.....	.....	.787	*.759*	.....	.....	.839	.830	.....	.....
4437.1	.....	.....	.....	.....	.152	.....	.102	.....	.....	.....	.....	.....	.....	.051	.....	.....	.....
4437.9	.....	.....	.....	.....	.935	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
4442.529	.548	.556	.531	.529	.506	.532	.528	.....	.531	.538	.531	.531	.594	.....	.544	.594	.500
4443.2	.254	.251	.237	.207	.209	.252	.182	.....	.224	.212	.240	.185	.203	.....	.212	.194	.....
4443.9	.975	.935	.949	.998	.966	.148	.977	.....	.139	.060	.052	.059	.181	.058	*.066*	.048	.....

4445.6	.....	.....	.....	.....	.....	.....	.668	.734	.....	.654	.684	.....	.....	.593	.....	.....	.....	.....
4447.2	.....	.....	.....	.....	.214	.....	.161	.....	.....	.....	.189	.....	.....	.....	.....	.....	.....	.....
4447.915	.950	.936	.971	.911	.919	.952	.928	.881	.959	.897	.892	.874	.930	.897	.879	.908	.881	
4450.7	.724	.722	.743	.749	.746	.746	.810	.749	.713	.767	.763	.716	.752	.750	.731	.720	.715	
4451.8	.....	.....	.901	.....	.....	.....	.....	.....	.....	.878	.....	.....	.....	.....	.....	.827	.....	
4454.9	.....	.....	.923	.....	.....	.....	.....	.964	.....	.....	.....	.....	.....	.955	.....	*.851*	.....	
4457.6	.....	.....	.659	.....	.....	.....	.....	.625	.....	.....	.....	.....	.....	.....	.....	.....	.....	
4458.3	.443	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.300	.....	
4459.3	.348	.342	.370	.356	.339	.351	.356	.323	.320	.314	.332	.324	.360	.290	.280	.281	.307	
4460.4	.468	.392	.369	.....	.430	.....	.466	.397	.....	.420	.318	.428	.....	.446	.....	.345	.345	
4471.1	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.182	.....	
4476.2	.226	.216	.208	.200	.211	.208	.227	.214	.176	.164	.184	.133	.182	.178	.191	.158	.182	
4481.4	.425	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	
4482.4	.470	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.456	.....	.....	.....	

NOTES

λ 4250.2 Mean, .292$_{13}$.
4250.9 Mean, .967$_{11}$.
4271.322 λ is 0.2 A lower than in types.
4276.84 A rather poor line.
4282.580 Mean, .582$_{4}$.
4284.5 Very broad blend.
4285.617 Mean, .610$_{4}$.
4292.307 Mean, .357$_{13}$.
4295.0 Broad.
4296.02 A slight variation indicated.
4296.8 Blend.
4298.1 λ slightly smaller near minimum?
4306.924 Mean, .926$_{14}$.
4308.041 Mean, .036$_{4}$.
4316.2 Narrow.
4319.7 .877 error? "v. faint and narrow."
4322.6 Narrow, faint. Mean, .676$_{8}$.
4325.1 Fuzzy.
4325.9 Fuzzy field to red.
4327.16 Mean, .124$_{11}$.
4328.104 Mean, .106$_{14}$.
4329.1 Faint, broad.
4330.7 Very broad, double, resolved. On Plate 1220 the two lines are of about equal intensity; on 1090 the component toward the red is the stronger, as it is also in the sun.
4332.9 Mean, .904$_{10}$.
4336.4 Faint.
4337.2 Red edge fuzzy. Faint companion included in measure near maximum?
4339.713 Mean, .683$_{6}$.
4341.5 Corresponds to G5.
4342.3 Mean, 341$_{9}$.
4349.14 Mean, 134$_{10}$.
4349.9 Faint. Mean, .957$_{9}$.
4352.0 Mean, .020$_{11}$, corresponds to G4.4.

λ 4356.12 Mean, .126$_{4}$.
4358.8 Broad, double. Possibly not always same components included in measure.
4360.9 .853 is blend with Ti.
4366.3 Very broad blend.
4367.9 Red edge fuzzy.
4369.9 Double, violet component weaker.
4376.1 Mean, .111$_{14}$.
4379.3 Mean, .382$_{9}$.
4387.0 Constant? Mean, .955$_{14}$.
4389.4 Mean, .403$_{11}$.
4391.14 Mean, .120$_{14}$.
4396.01 Mean, .032$_{10}$.
4401.613 Mean, .614$_{14}$.
4404.932 Mean, .924$_{13}$.
4406.81 Mean, .781$_{6}$.
4408.579 Much stronger at minimum and in sun-spots.
4409.5 Mean, .539$_{8}$.
4411.0 Mean, .007$_{8}$. "Broad; Double?"
4413.7 λ may diminish toward minimum.
4415.4 Strong, broad (double). Variation may be real.
4420.694 Varies? Mean, .694$_{14}$ corresponds to G4.6.
4423.0 Broad.
4424.3 Not always same components measured.
4430.1 Corresponds to K0 type.
4430.7 Correspond to G8 and K2 respectively.
4435.4 For mean of 35.2 and 35.7.
4437.9 Broad and strong at minimum. At maximum very faint.
4445.6 Mean, .669$_{3}$.
4447.915 May vary. "Winged" in spots.
4450.7 Broad? Mean, .742$_{14}$.
4460.4 Varies? Mean, .390$_{8}$.
4471.1 Broad; fades off on edges.
4476.2 λ varies in *l* Carinae and in types.
4482.4 Much stronger at minimum. "Much widened" in spots.

PART II. CHANGES OF SPECTRAL TYPE IN l CARINAE

My early measures and reductions, in 1908, showed quite clearly changes of spectral type from maximum to minimum light. This change of type is at once evident by comparing under a microscope a plate taken near maximum with one taken near minimum. Unfortunately the spectrograms are not at hand, so that I shall have to be content with reproducing some of my notes from such a comparison made rather hastily in 1908:

The absorption is greater at light-minimum. The spectrum has a somewhat knotted appearance, giving the impression of the presence of bright lines. Probably on account of the increased absorption this appearance becomes more prominent at minimum. A direct comparison of plates 1115 II and 1220 IV, at 2.6 and 24.0 days respectively after maximum, shows:

4351 stronger at minimum.
4352.9 stronger at minimum.
4358.9 widened toward the violet and stronger at minimum.
4376.1 faint at maximum; strong at minimum.
4404.9 stronger and widened at minimum—thus resembling late type.
4408.5 becomes very much stronger at minimum. Compare appearance with 4409.5.
4413.7 good and sharp at maximum; becomes somewhat weaker and fuzzy at minimum.
4422.0 widens toward violet at minimum; violet edge fuzzy at minimum.
4325.6 red edge fuzzy at maximum; line broad, strong, and sharp at minimum; very marked.
4437.9 broad and fairly strong at minimum; at maximum very faint.
4445.7 strong and sharp at minimum; very faint (not measurable) at maximum.
4471.0 has edge toward red fuzzy at maximum; sharp at minimum; very decided difference.
4481.4 Mg and Ti much weaker at minimum.
4482.3 much stronger at minimum.
4490 much stronger at minimum.

The last line is a blend of 4489.91 Fe 4, in sun-spots 6, and 4490.25 Mn–Fe 3 N, in spots 3–4. These notes are especially interesting when compared with the data for the same lines in Tables II and III, most of which were subsequently obtained and which they supplement as far as they go. The changes of intensities from near light-maximum (type F9.4) to near minimum

(type G8.6)[1] run parallel with the changes from sun (type G) to sun-spots (type K). Note also, for example, that 4445.7, marked "very faint (not measurable) at maximum," later received no measures near maximum.

Although the wave-lengths resulting from my early reductions were at the time compared for type with my first curves of variation in the stellar series, publication of the results was delayed because this comparison lacked the quantitative accuracy which was readily foreseen to be possible. The improvement of the curves for the stellar types and my method for determining stellar types[2] resulted largely from a desire to determine accurately the changes of spectral type in variable stars in a way which would be strictly quantitative and be as free as possible from the personal element. The results given below for *l* Carinae still do not represent the greatest accuracy attainable by these methods, for the reason that the measures are not all by one observer, and, from the experience gained, a somewhat improved list of lines could be selected for measurement. I hope that observers with adequate instrumental equipment may extend this work to other variables and to other regions of the spectrum, as we may confidently expect very profitable results. If desired, I would gladly suggest a suitable list of lines to anyone who wishes to take up a similar study. The dispersion of three prisms should be employed in order to have the scale sufficiently large. The effects noted below were first looked for on the one-prism spectrograms of Y Ophiuchi and T Vulpeculae, but the dispersion was inadequate. For best results all the different factors entering into the problem, such as slit-width, general intensities of exposure, development, etc., should be maintained as constant as possible. Except for very short light-periods, the exposure time may more safely be varied than the slit-width to allow for the changing intensities of the light. For the present, in order to avoid certain systematic effects, discussed in a report to the Solar Conference in 1913, the curves of variation of wave-length upon which determinations of type are to be based

[1] See Table IV, Plates 1115 and 1220.

[2] *Astrophysical Journal*, **33**, 130, 1911.

Lines of Varying Wave-Lengths

Spectrograms arranged according to phase of light variation

Continuous lines for *l* Carinae; dashed lines for types

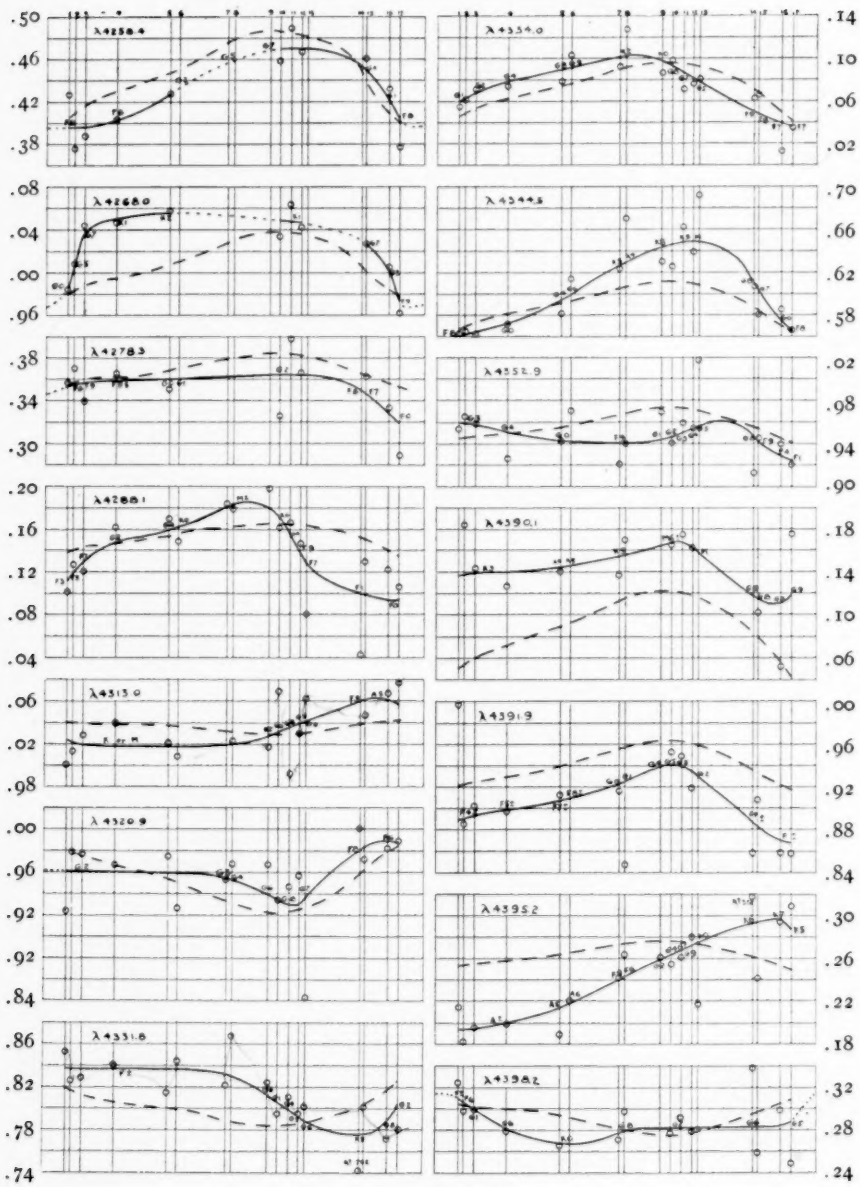

FIG. 1

Lines of Varying Wave-Lengths—*Continued*

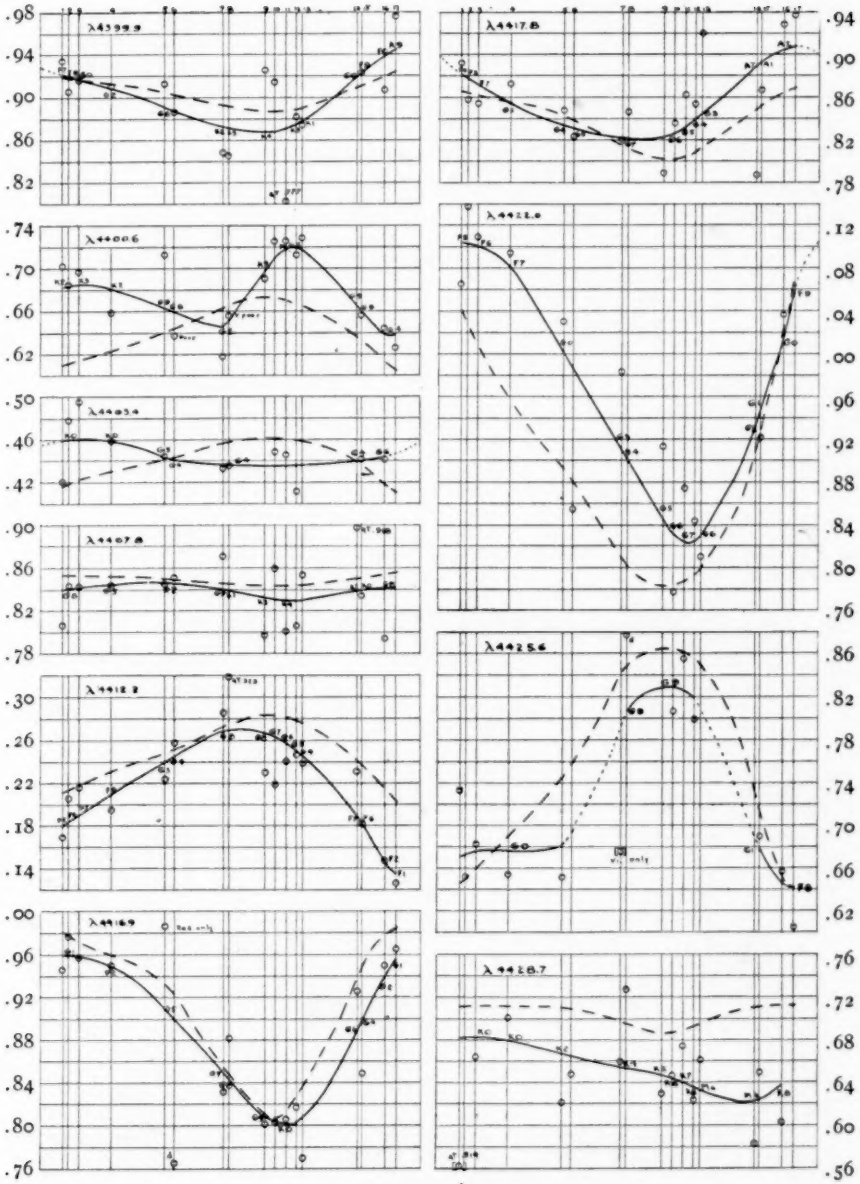

FIG. 1—*Continued*

Lines of Varying Wave-Lengths—*Continued*

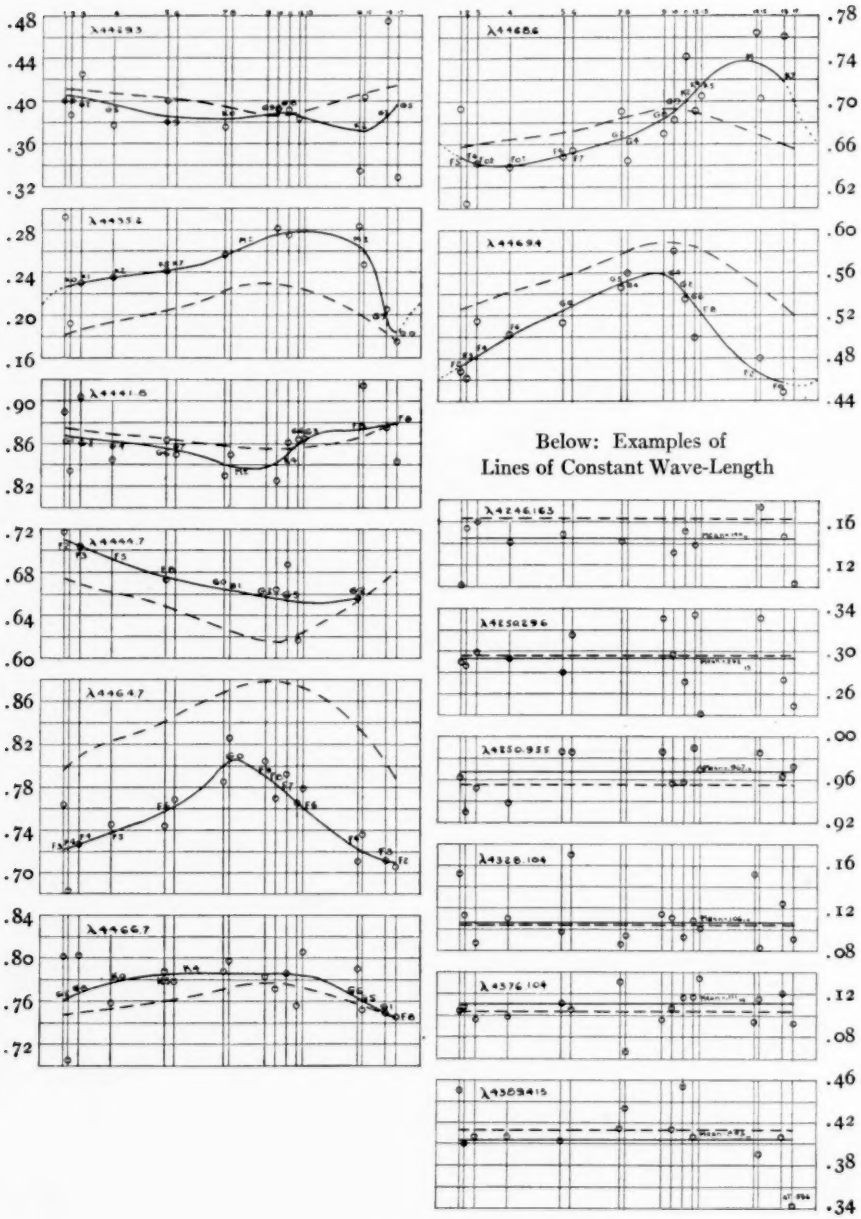

FIG. 1—*Continued*

should be determined from plates taken with the same instrumental equipment which was used to obtain the plates for the star under consideration.

The wave-lengths in Tables II and III furnish the data for a quantitative determination of the spectral type of *l* Carinae corresponding to each spectrogram. The individual lines give separate determinations of type by comparison of the observed wave-lengths with the curves of variation found for the lines in the general series of stellar types. With but two exceptions only curves published in *Boletin No. 1* of the Córdoba Observatory (1911) were used. The results of this comparison are collected in

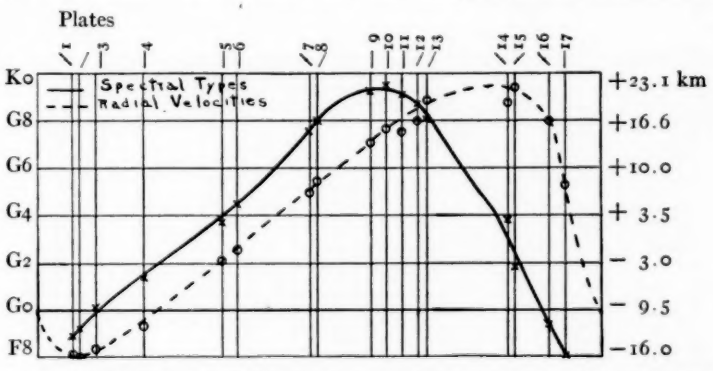

FIG. 2.—Simultaneous variations of spectral type and of radial velocity in *l* Carinae.

Tables IV and V, which are self-explanatory, and in the figures. The weights in the third column of Table IV were formed according to (*a*) the range of variation of wave-length from types F to K in the stellar types, and (*b*) the definiteness of the curve showing the change of wave-length with change of phase in *l* Carinae, taking into account also the number of observations upon which the curve is based. The means at the bottom of Table IV, which may be regarded as fairly representing the actual changes of type in this star, are compared in Figure 2 with the simultaneous changes of radial velocity. The curves for spectral type and for radial velocity in the figure synchronize with each other about as closely as other variations in Cepheids have been found to synchronize with each other.

TABLE IV

THE TYPES INDICATED BY THE INDIVIDUAL LINES FOR EACH SPECTROGRAM OF *l* CARINAE

λ	No. of Obs.	Weight	312 1	1115 2	1120 3	1124 4	1147 5	1149 6	1040 7	299 8	1201 9	1209 10	1212 11	1220 12	1150 13	311 14	1079 15	1090 16	1098 17
4258.4	11	.50	F8	F8	F8	F8	G1.5	G2	G5	G5.5	G7	G7.5	G7.5	G7.5	G7.5	G5	G4	G1	F8
68.0	11	.50	G0	G5	G7	K1	K2	K2	K1.5	K1.5	K1	K1	K1	K1	K1	G8	G7	G3	F9
78.3	11	.34	F7.5	F8	F9	F9.5	G0	G1	G1	G2	G2	G2	G2	G2	G1	F8	F7	F0	F0
88.1	17	.45	F3	F5	F7	G2	G8	K0	M±	M±	M±	K6	G4	F9	F7	F1	F0	F0	F0
4313.0	15	.0	(K or M)								K2	K0	G8	G5	G0	F9	A5	A5	A5
20.9	17	.45	G2	G2	G2	G2	G2	G3	G3	G4	G6	G8	G8	G8	G7	F8	F8	F6	F0
31.8	17	.0	(F±)								G0	G1	G4	G5	G6	K5	K5	G7.5	G2.5
34.0	17	.50	G1	G1	G2.5	G4	G7.5	G9	K2	K2	K0	G8	G7	G5	G5	F9	F8.5	F7	F7
44.5	17	.50	F8	F8	F8	F9.5	G4	G6	K3	K4.5	K8	K8.5	K9	M0	M0	G8.5	G7	G0	F8
52.9	17	.38	G5	G5	G5	G4	G0	G0	F9	F9	G1	G2	G3	G4	G5	G2	F9	F4	F1
90.1	12	.38	K3	K3	K3	K3	K4	K5	K9	Ma	Mb	Mb±	Mb±	Mb±	M0	G9	G8	G8	G9
91.9	14	.30	F4±	F4±	F4±	F5±	F7±	F8±	G0	G1	G4.5	G5	G5	G2.5	G2	F4±	F4±	F±	F ±
95.2	17	.34	A±	A±	A±	A±	A5	A6	F4.5	F6	G2	G3.5	G7	G9	K0	K0	K0	K7	K5
98.2	14	.38	F5	F6	G1	G6.5	K0	K0	G8	G8	G8	G7	G7	G7	G7	G6	G6	G6	G5
99.9	16	.42	F7	F7.5	G0	G2	G7.5	G8	K2	K3	K3.5	K4	K3.5	K3	K1	G0	F9	F0	A9
4400.6	16	.29	K2.5	K2.5	K3	K2	G9	G8	G5.5	G5.5	K9	Mb±	Mb±	Mb±	Mb±	G9	G8	G3.5	G3.5
03.4	12	.40	K0	K0	K0	G9.5	G5	G3.5	G3.5	G3.5	G3.5	G3.5	G3.5	G3.5	G3.5	G4	G4	G5.5	G6
07.8	15	.16	G8	G8	G8	G5	G2	G2	G9	K1	K3	K3.5	K3.5	K3.5	K3.5	K1	K0	G8	G8
12.2	17	.50	F5	F6	F7	F9	G3	G4	G8	G8	G8	G7	G6	G5	G4	F7	F6	F2	F1
16.9	17	.45	G1	G1	G1	G2	G5	G5	G7.5	G8	G9	G9.5	G9.5	G9.5	G9.5	G6.5	G4.5	G2	G1
17.8	17	.45	F0	F3	F7	G1	G4.5	G5	G7	G7	G7	G6	G5	G4	G3	A7	A1	A0±	A0±
22.0	14	.50	F5	F5	F6	F7	G0	G0.5	G3	G3.5	G5	G6	G7	G7	G6	G2.5	G1.5	G0	F9
25.6	13	.33	G0	G0	G0	G0	G0	G0	G5	G9	G9	G9	G9	G9	G6	G3	G1	G0	F8
28.7	14	.27	K0±	K0±	K0	K0.5	K2	K2.5	K3.5	K4	K5	K6	K7	K8	Ma	Mb	Mb	K8	K5±
29.3	12	.27	G0	G0	G1.5	G5	G9	G9	G9.5	G9.5	G9	G8	G8	G8	G8	K2	K2	G7	G5
35.2	12	.45	K0	K0	K1	K2	K5	K7	M±	M±	M±	M±	M±	M±	M±	M±	M±	G3	G0
41.8	15	.16	G1	G1	G2	G4	G6	G7	M±	M±	M±	M±	K4	G6	G3	F8.5	F8	F8	F8
44.7	7	.41	F2.5	F2.5	F3	F5	F8	F8.5	G0	G1	G2	G2	G3	G3	G3	G2	G2	G0	F7
04.7	17	.50	F3.5	F3.5	F4	F4.5	F6	F6	F9.5	G0	F8.5	F8	F7	F7	F6	F4	F3.5	F3	F2.5
06.7	17	.38	G5	G5	G7.5	K0	K3	K4	K4	K4	K4	K4	K4	K4	K4	G6	G5	G1	F8
08.6	16	.38	F5	F4	F0±	F0±	F4	F7	G2	G4	G8	G9	K2	K4	K5	Mb	M0	K7	K0±
09.4	12	.50	F2.5	F3	F4	F6	F9.5	G0.5	G3	G4	G4	G4	G1.5	G0.5	F8	F2	F1.5	F0	F0
12.9	14	.41	F6	F6	F6	F7	G1	G1	G5	G5	K4	K5	K6	K6	K6	K3±	G7±	G6±	F9
Weighted means			F9.0	F9.4	G0.2	G1.5	G3.7	G4.4	G7.5	G8.1	G9.3	G9.4	G9.0	G8.6	G8.1	G3.8	G1.8	F9.5	F8.0
Sum of weights			12.2	12.2	12.2	12.2	12.2	12.0	12.0	12.0	12.2	12.2	12.2	12.2	12.2	12.0	1.20	12.0	12.2
Probable error for the mean			±0.9	±0.9	±0.9	±0.9	±0.8	±0.8	±0.8	±0.8	±0.8	±0.8	±0.8	±0.8	±0.8	±1.1	±1.1	±1.1	±1.0
Probable error of an observation of weight 1			±3.1	±3.0	±3.0	±3.0	±2.9	±2.7	±2.8	±2.7	±2.7	±2.7	±2.7	±2.9	±3.1	±3.7	±3.9	±3.8	±3.4

NOTES

λ 4313.0 Partly indeterminate—range from F to K only .020 A.
4331.8 Partly indeterminate.
4390.1 Questionable whether this line should be used here.
4391.9 Partly extrapolated. Weight too high?
4403.4 Direction of variation seems opposite to that in types.

λ 4422.0 Compared with unpublished curve.
4428.7 Curve systematically too low by .03A±.
4435.2 Curve systematically too high.
4441.8 Compared with unpublished curve.
4472.9 1149, 1040, 299, 311, 1079, and 1090 have weight 0.16.

NOTE.—In Table IV, the seventeen vertical columns show a range of types one and three-fourths times as great as the horizontal lines. On account of the preliminary status of the method some lines were included in the determinations of type for which slight changes of wave-length correspond to large changes of type. The inevitable result is that not only will moderate accidental errors of measurement increase both the horizontal and the vertical ranges greatly, but small bodily displacements of wave-length for such a line in the spectrum of the star will shift the corresponding changes of type to a very different level, directly affecting the range in the vertical columns. To illustrate: One line may show a range of variation from, let us say, A to G while another line may indicate variations between G and K or M, the variations for the two lines synchronizing with each other and with the light- and velocity-variations of the star. For these reasons the probable errors accompanying Table IV, which were computed from the vertical ranges actually listed, do not represent the limit of accuracy which is inherent in the method. The means at the bottom of the columns may nevertheless be regarded as representing fairly well the actual changes of type of the star, because the errors referred to above, which are systematic for individual lines, partake of the nature of accidental errors for purposes of the means.

In order that the reader may not carry away the erroneous impression that an enormous amount of time will be involved in practical applications of the method employed above for the determination of spectral type, it may be well to call attention to the fact that the method is still in the preliminary stages of development. The aim has been to include as large a number of lines as the nature of the spectra and other limitations would permit, for the simple reason that the behavior of all lines was unknown and could not be safely determined by inference. As was pointed out in the article, a considerable amount of exploratory work is still urgently needed. Eventually measurement for type will be restricted to a very limited list of most suitable lines, probably not more than from eight to twelve per plate in addition to those used for radial velocity.

TABLE V

ADDITIONAL LINES GIVEN AS VARIABLE IN *Córdoba Observatory Boletin No. 1*

λ 4254.5	Partly indeterminate and partly discordant.
4267.1	Curve for stellar types is too weak—only six observations.
4274.9	Unsatisfactory. Not entirely accordant, nor is it especially discordant.
4293.2	Too few observations in *l* Carinae.
4314.3	Partly in accord in *l* Carinae, but the small range, together with the oscillations in stellar types, render this line unsuited for determinations of spectral type.

4315.1 Wave-lengths in *l* Carinae are nearly all above the highest parts of the curve for stellar types. This is one of the few lines which seem to be discordant.

4318.8 Partly accordant and partly indeterminate. The range of variation is apparently greater in *l* Carinae than in stellar types.

4321.9 Partly indeterminate—range from G to K2 is only .016 A.

4325.1 Wave-length changes by approximately the same amount as in types G to K, but the curve for types is systematically higher by .035 A±.

4325.9 More or less accordant, aside from being systematically higher by .035 A±.

4340.6 Hγ. One measure for Plate 1212 corresponds to type K2.

4351.1 Weakly determined both in types and in *l* Carinae.

4352.0 Not suitable for this purpose with present data—variation only .011 A from F to K.

4354.7 Data weak in *l* Carinae and in types.

4359.8 More or less indeterminate in types F to K except with a large number of observations.

4362.2 Data weak.

4394.2 In types later than G the measured line is a blend of λ .225 with two weak companions to the violet. The former gradually becomes weaker and the latter stronger. In *l* Carinae the faint companions were not included in the measures, as is indicated by a note for measure on Plate 1220.

4427.4 Omitted because too much of results had to be extrapolated.

4443.9 The measure sometimes includes the companion to the red, λ 4444.385. The displacement toward the red in the later types and in *l* Carinae near light-minimum may be largely due to personality in the settings on close pairs; the companion at λ .385 is then moderately strong, whereas it is quite weak in early types and in *l* Carinae near maximum light. With the additional data which I have for stellar spectra taken with several different dispersions, it should be possible to separate the personality effects from the true displacements.

4459.3 Not suited for this purpose, because in stellar types the variation of the wave-length is confined almost entirely to types earlier than G.

The Figures 1, in which the observed wave-lengths on individual spectrograms are represented by circles, give three things for each line: (*a*) the curve showing the variation of wave-length in *l* Carinae with change in phase of velocity and variation of the light; (*b*) the spectral type for each plate obtained by comparing the wave-length taken from the curve in (*a*) with the curve for the stellar series;

and (*c*) that portion of the curve for the stellar series which corresponds to the means of the types for the individual spectrograms of *l* Carinae, i.e., from F9 to G9.4 and back to F8.

The two curves in the graphs generally rise or fall together toward light-minimum, even when widely separated by systematic differences, thus illustrating how the individual lines show the progressive change of the star to the so-called later types. Note especially the lines with a large range of variation, such as 4412.2, 4416.9, 4422.0, 4425.6, 4464.7, and 4469.4. For only one line, 4403.4, the main tendencies of the two curves indicate opposite directions of change of wave-length. Minor or secondary deviations between the two curves are to be expected, first, because the present data are not ideal in every way, as explained above, and second, because of the fair possibility—and even perhaps probability—that the changes of type with changes of phase in the light and variations of velocity do not coincide in every detail with the corresponding stellar types as classified at present, and, moreover, may not always repeat themselves exactly in different periods. Also in the variable star where the surface conditions—i.e., in the comparatively shallow layer in which all of the light that reaches us originates—are undergoing constant and apparently violent changes, it would seem that a considerable overlapping of the distinguishing characteristics of types almost certainly takes place. An exact correspondence would perhaps be more surprising than the presence of moderate differences.

An interesting comparison is to follow in detail (for the components of lines) the changes of intensity from sun to sun-spots and from spark to arc and to compare these with the observed changes of wave-length in the series of stellar types and in *l* Carinae. The last five columns in Table II aid in showing the remarkable extent to which all of these changes run parallel. Some of these have been collected in Table VI in order to bring out, in a way in which verbal description cannot, how some of the shifts are produced by a weakening of an enhanced component as the star approaches light-minimum, while other shifts are due to a marked strengthening of arc or of sun-spot lines. "Str" stands for a strengthening and "wk" for a weakening of the component.

TABLE VI

λ_{Rowland}	Sun	Sun Spots	Spark	Arc	Near Minimum		
					Violet Comp.	Red Comp.	Shift of Blend in Star
	a) Enhanced Lines or Weakened in Spots						
4261.891	2	.....	.....	.....	.....	.....	.....
.086	1	o	Cr 5–6	o	.....	wk	to violet
.142	1						
4314.248	Sc 3	2	.....	.....	wk	.....	to red
.381	1						
.479	Ti 1						
.673	ON						
4320.661	o	1–2	.....	.....	.....	.....	.....
.756	oo						
.907	Sc 3	3					
.119	2	1	Ti 3	1	.....	wk	to violet
4371.3	.....	.....	.....	.....	wk	.....	to red
4395.201	Ti 3	2	Ti 9	5	wk	.....	to red
4399.776	Ni o	.....	.....	.....	.....	.....	.....
.935	Ti, Cr 3	2	Ti 7	3	.....	wk	to violet
4412.092	1	o	.....	.....	wk	.....	to red
4416.636	V o	1:1	.....	.....	.....	.....	.....
.811	ooN						
.985	2	1	.....	.....	.....	wk	to violet
4469.316	Ti 1	o–1	.....	.....	wk	.....	to red
.441	oooo						
.545	Fe 4	4					
4472.884	Fe 1	2	.....	.....	.....	.....	.....
.967	Mn o						
.096	Ni? o	oo	.....	.....	.....	wk	to violet
	b) Spot Lines or Arc Lines						
4258.219	1N	o–1	.....	.....	.....	.....	.....
.477	Fe 2	4	.....	.....	.....	str	to red
4265.832	Ti o	o–1	.....	.....	str	.....	to violet
.081	Mn 2	2–3					
4278.308	o	.....	.....	.....	.....	.....	.....
.390	Fe-Ti 3	3–4	.....	.....	.....	str	to red
4288.038	Ti 2	.....	.....	.....	.....	.....	.....
.149	Ni 1	.....	Ru 1	1			
.310	Ti, Fe 1	2	.....	.....	.....	str	to red
4321.813	Ti o	1	.....	.....	str	.....	to violet
.961	Fe 2						
.204	ON						
4344.451	Ti 2	n.c.	Ti 3	1	.....	.....	.....
.670	Cr 4	6	.....	.....	.....	str	to red
4351.000	Ti 1	n.c.	Ti 2	o	.....	.....	.....
.216	Cr 3	5	.....	.....	.....	str	*
4371.144	Zr 1	1–2	Zr 3	1	.....	.....	.....
.221	1						
.320	oo						
.442	Cr 2	3–4	.....	.....	.....	str	to red

* To red in late types.

TABLE VI—*Continued*

λ_{Rowland}	Sun	Sun Spots	Spark	Arc	Near Minimum		
					Violet Comp.	Red Comp.	Shift of Blend in Star
	b) Spot Lines or Arc Lines						
4389.801	o	.....	.....	.....	.....	.....	.....
.930	Mn, Ni? o						
.034	oo						
.149	V 2	4	.....	.....	.....	str	to red
4395.201	Ti 3	2	Ti 9	5	.....	.....	.....
.413	V, Zr 2	3	.....	.....	.....	str	to red
4400.343	Zr ond?	.....	.....	.....	.....	.....	.....
.555	Sc 3	3–4					
.738	V 1	3	.....	.....	.....	str	to red
4407.810 s	V 2	8	.....	.....	str†	.....	to violet
.871 s	Fe 4						
4412.092	1	o	.....	.....	.....	.....	.....
.297	V oo						
.415	Cr o	1:o	Mo 2	o	.....	str	to red
4416.636	V o	1:1	.....	.....	str	.....	to violet
.811	ooN						
.985	2	1					
4421.733	V o	1:1	.....	.....	str	.....	to violet
.928	Ti oo	o	Ti 3	2			
.104	1	oo–o					
.226	o						
4464.617	Ti? 2	n.c.	Ti 3	1	.....	.....	.....
.844	Mn 2	3–4	.....	.....	.....	str	to red
.938	Fe 1						

† Probably V is strengthened relatively more than Fe.

For numerous lines where the observed changes are clearly in the direction accordant with changes from sun to sun-spots, no application could be made to the determination of type because of inadequate data in stellar types. Nevertheless these lines also serve to confirm the fact that changes occur in the spectrum of *l* Carinae which are in many ways similar to consecutive changes of spectral type.

PART III

Recently Adams and Joy[1] have noted a marked distinction in the spectral types of Cepheids as obtained from the hydrogen lines alone or from the general spectrum. For nine stars they find in the mean a spectrum of F1 at maximum and F7 at minimum of light, based on the hydrogen lines alone. From the general

[1] *Proceedings of the National Academy of Sciences*, **4**, 129–132, 1918.

spectrum they obtain for the same stars F9 at maximum and G0 at minimum. The mean variation of type indicated by the hydrogen lines is 0.6 of the interval between F and G, whereas the general spectrum shows a variation of only 0.1. That is, according to Adams and Joy, the general spectrum shows very little change of type during the period of light-variation. These observers note, however, in addition to the changes in the hydrogen lines, certain other differences in the spectra of Cepheids at maximum and minimum, the three most important of which, also previously observed, are: (1) a shift of the maximum of the continuous spectrum toward shorter wave-lengths at maximum of light; (2) a general, slight widening of the spectral lines at minimum; (3) an increase in the intensity of the so-called "enhanced" lines at maximum. The first of these can influence the positions of the lines only indirectly and in a minor way, and need not be especially dwelt upon here. The second and third will be referred to again later.

The main result of Adams and Joy, namely, that the indications of changes of spectral type in the Cepheids are confined chiefly to the hydrogen lines, appears to be completely at variance with my observed changes of spectral type in *l* Carinae, as determined from the general spectrum and given in detail above. In view of these contradictory results it may be well to discuss the data in Part II in somewhat greater detail, with special reference to their bearing upon this question.

The methods employed in Part II for the determinations of spectral type are essentially different from those employed by Adams, Shapley, and Joy. The latter are based upon differential relative changes in the intensities and widths of lines, partly estimated and partly measured. The former were intentionally freed as much as possible from qualitative estimations by placing the entire weight in the determinations of type upon the measured positions of the lines. Each of these methods has its limitations as well as its advantages, and I am inclined to the view that the best results will be attained by combining the two methods. To me it seems quite unsafe to neglect the measured positions of the lines, as these furnish the best means of identifying with a fair degree of certainty the components involved in observed changes of intensity and width.

As, unfortunately, the *l* Carinae spectrograms are in Santiago, they cannot be referred to at this time with the special aim of directly bridging the gap between the general spectrum and the hydrogen spectrum. My notes state that "in κ Pavonis and *l* Carinae (F5 and G types) the Hγ region looks very fuzzy and diffuse, Hγ being v v broad"; and they also contain the suggestion to "see if Hγ gets narrower and sharper toward light-minimum" than at maximum. This suggestion was not followed up because of other demands upon my time. Evidently the failure to measure Hγ on sixteen of the seventeen spectrograms is attributable to poor quality of the line, possibly in part due also to interference of close neighboring lines and to underexposure of the Hγ region on some of the spectrograms.

The lines observed, given in Tables II and III, belong to and are among the most characteristic and important lines in the general spectrum. Their measured positions identify them, on the whole I believe without any serious outstanding uncertainties, with corresponding lines in the spectra of the sun and of sun-spots, and these in turn have been identified with corresponding lines in stellar spectra. The lines having constant wave-lengths in the stellar types also have constant wave-lengths in *l* Carinae. The lines which vary progressively during the period of variation of light (Table IV), and upon which the changes of spectral type are based, are among the most important lines showing progressive changes of wave-length in the series of stellar types. Among the identified components of these lines are Cr, Fe, Mn, Sc, Ti, and V, elements of which the spectral lines exhibit marked and progressive changes of intensity in types A to M.

Nor are the observed changes in *l* Carinae covered by the general widening of spectral lines at minimum and by the increased intensity of the "enhanced" lines at maximum, effects (*b*) and (*c*) referred to by Adams and Joy. Table II shows numerous lines for which the position of the blend is progressively shifted in *l* Carinae from maximum to minimum light and in stars from early to late types, in the direction indicated by the changes in the intensities of the components from sun to sun-spots. Note, for example, the lines 4266.0, 4296.0, 4320.9, 4416.9, and 4422.0, for which both the shift indicated by sun-spot data and the observed shift in *l* Carinae

and in the later stellar types are to the violet; and the lines 4278.3, 4288.1, 4390.1, 4391.9, 4412.2, and 4469.4, for which the indicated and observed displacements are to the red. For most of these lines no enhanced components are involved. Several lines, 4292.3, 4298.1, 4391.1, and 4408.5, for which the sun-spot data indicate balanced changes for the intensities of the components, have practically constant wave-lengths in stellar types. Some lines, like 4334.0, show clearly accordant variations in *l* Carinae and in types, with no spot data given. Perhaps corresponding changes will be found in spot spectra. The wholly or partly discordant lines are few. Thus, for 4277.6 the observed displacement is toward the violet both in types and in *l* Carinae, contrary to the expected shift. For 4420.6 a slight shift toward the red might possibly be expected; the observed shift in the variable star is to the violet; in types the data are weak. Such apparent discordances may later be removed in part by additional data from spots and Cepheids. On the whole, the agreement between the expected or inferred and the observed changes of wave-length of the blends is remarkably close.

The "general slight widening of the spectral lines at minimum," additional effect (2) of Adams and Joy (above), seems also brought out in my notes of 1908, reproduced in Part II—unless I misconstrue the intent of these observers. Can the greater part of the apparent widening be explained as a widening of blends due to a considerable increase in intensity of one or more of the components rather than as a genuine widening of single lines? As in later types, so also in *l* Carinae toward minimum of light, the selective absorption increases, many more components being strengthened than weakened, with the result that numerous blends are apparently widened and a small number perhaps narrowed. This, however, does not explain all increases in width, as, for example, for 4468.6, which is apparently a single line in the sun.

My data also contain partial evidence bearing upon Adams and Joy's effect (3)—"an increase in the intensity of the so-called enhanced lines at maximum"—which seems in part confirmatory and in part suggests the changed statement "a greater relative intensity of the so-called 'enhanced' lines in *l* Carinae than in the corresponding types in the stellar series." For the former state-

ment the two curves in Figures 1 for the lines having enhanced components should diverge more and more as maximum of light is approached, the relative shift in *l* Carinae being in the direction of the enhanced component of the blend. This seems actually to occur for one or two lines (see 4395.2). For other lines (see 4464.7) the two curves remain parallel throughout their entire length, the systematic displacement between them being in the direction required for greater strength of the enhanced component in the variable star than in the corresponding stellar types. However, the evidence is not clear-cut either way. And, besides, for several lines for which the two curves are parallel and separated no enhanced components are given.

As one of the results of the foregoing investigation it is possible to outline roughly an extensive field of research which can profitably be followed. In order to be able to distinguish the characteristics shared by all Cepheids from those peculiar to individual stars, the spectra of a number of Cepheids should be studied in greater detail. The digressions from a strictly parallel behavior of the lines in Cepheids and in the series of stellar types may lead to results more interesting and far-reaching than the accordances. Also the emissions, the "knotted appearance" referred to in my notes, deserve further study. It may be to the point to recall that the one fact which more than any other led to serious questioning of the binary-star explanation for Cepheids was the discovery of the apparent relation between the light- and the velocity-variations. With dispersions equal to and greater than that of the three-prism Mills spectrographs, it should be possible to follow the simultaneous changes of the lines for the different elements through the cycle of changes of light and velocity.[1] Such results for a number of Cepheids would provide the necessary data for correlating any effects found with such factors as (1) the length of the period; (2) the limiting types between which the variations take place; (3) and (4) the types and the range of variation indicated by the different classes of lines (i.e., spark, arc, spot, etc.), and by the lines of individual elements or groups of elements; and (5) more

[1] The data in Part I are not sufficiently complete to permit of doing this in ample detail for *l* Carinae.

exact relationships of the curves representing variations of visual light, photographic light, velocity, color-index, and type.

Thus even the meager data at present available seem sufficient to establish a relation with the length of the period of light-variation. This seems to have been observed by Campbell[1] and later by Shapley[2] to apply to Cepheids as well as to the general list of variable stars as found, I believe, first at Harvard. However, the exact relationship noted below seems not to have been brought out. Table VII, taken in part from Shapley[3] and Seares and Shapley,[4] with *l* Carinae added, shows a fairly definite progression of the Cepheids toward "later" types, see columns "Shapley" and "Albrecht," with increasing length of the period. This effect seems more clearly and definitely shown in the means of groups of two or three consecutive stars, as given at the bottom of the table. Apparently it is much more strongly marked for very short periods and diminishes to slight changes for periods greater than five days. The poorly marked, slow increase in the length of the period with spectral type, found by Campbell, is thus readily accounted for by the poor representation of very short periods in Campbell's tables. The effect seems absent for the hydrogen spectrum in Adams and Joy's list of nine stars (*loc. cit.*) and to be indicated only slightly in their results from the general spectrum. Here also its apparent absence or small value seems to lie in part in the small number of stars used and in part in the absence from their list of stars with very short periods.

In Table VII my types for *l* Carinae should perhaps be somewhat reduced to make them strictly comparable with the types found by Shapley, due to the presence of a second effect, namely, that the hydrogen lines yield somewhat "earlier" types than the general spectrum. This second effect, which is apparently shown in the tables of Adams and Joy (*loc. cit.*) and somewhat strengthened by my results from the general spectrum of *l* Carinae, seems to

[1] *Lick Observatory Bulletin*, **6**, 51, 1910.

[2] *Astrophysical Journal*, **40**, 451, 1914, and *Contributions of the Mount Wilson Observatory*, No. 92.

[3] *Ibid.*, **44**, 274, 1916, and *Contributions of the Mount Wilson Observatory*, No. 124.

[4] *Ibid.*, **48**, 238, 1918, and *Contributions of the Mount Wilson Observatory*, No. 159.

TABLE VII

Star	App. Mag. at Max.	Period (Days)	Change in Color-Index				Types (at Max.—at Min.)				Type Range (Shapley, Albrecht)
			Observed Photometrically	From Spectrum			From Hydrogen		From General		
				Hydrogen		General					
				Adams and Joy	Shapley	Adams and Joy or Albrecht	Adams and Joy	Shapley	Albrecht	Adams and Joy	
RS Boötis	9.2	0.38						B8–F0			1.2
XZ Cygni	8.7	0.47	.58	(.32)			A0–A8	A0–A6			0.6
RR Lyrae	6.8	0.57						B9–F2			1.3
SU Cassiopeiae	5.9	1.95	.14	.12	.20	.04	F3–F6	A9–F4		F7–F8	0.5
TU Cassiopeiae	7.2	2.14						F1–F8			0.7
SZ Tauri	7.2	3.15						A9–F7			0.8
RT Aurigae	5.1	3.73	.35	.32	.40	.12	F0–F8	A9–F9		F7–G0	1.0
SU Cygni	6.2	3.85	.35		.36			A7–F6			0.9
T Vulpeculae	5.5	4.44	.40		.44			A9–G0			1.1
δ Cephei	3.5	5.37	.49	.40	.48	.00	F0–G0	F0–G2		G0–G0	1.2
Y Sagittarii	5.4	5.77						F4–G4			1.0
X Sagittarii	4.4	7.01						F1–G5			1.4
U Aquilae	6.2	7.02						F6–G2			0.6
η Aquilae	3.7	7.18	.42	.32	.44	.08 .40	F1–F9	F0–G1	F6–G6	F9–G1	1.1 1.0
W Sagittarii	4.3	7.60						A8–G2			1.4
W Geminorum	6.7	7.92						F2–G1			0.9
U Vulpeculae	6.5	7.99	.4		(.24)			F8–G4			0.6
S Sagittae	5.5	8.38	.53		.32			F4–G2			0.8
ζ Geminorum	3.7	10.1	.38	(.04)		(.04)	F2–F3			F9–G0	
Y Ophiuchi	6.1	17.11						F5–G3			0.8
T Monocerotis	5.7	27.01						F5–G2			0.7
l Carinae	3.6	35.5				.46			F8–G9		1.1
Means for Groups											
3 stars		0.47						B9–A9			1.0
3 stars		2.41						F0–F6			0.7
3 stars		4.01						A8–F8			1.0
2 stars		5.57						F2–G3			1.1
3 stars		7.07						F2–G3			1.0
3 stars		7.84						F3–G2			1.0
1 star		8.38						F4–G2			0.8
2 stars		22.1						F5–G2			0.8
1 star		35.5						(F3–G4)*			1.1

* Estimated correction of −0.5 applied to make it systematically comparable with the values from the hydrogen spectrum. This correction, based on data of Adams and Joy and extrapolated beyond their limits of type, is quite uncertain.

have its source in the relatively greater strength of the hydrogen lines than of the lines of the general spectrum. In this connection it may be well to bear in mind also, as pointed out above, that the changes in the spectra of Cepheids are more complex than can be accounted for by pure and complete changes of type.

The range through which the type varies in Cepheids (see last column of Table VII) seems to be independent of the period, and to be nearly constant, averaging a little less than one type-interval (10.0 tenths).

Seares and Shapley (*loc. cit.*) have pointed out that the color changes in Cepheids inferred from the hydrogen spectrum (from the relation that the change in color equals 0.4 change in spectrum) agree with the color changes observed photometrically for these stars. On the other hand, the changes of color inferred from the general spectrum are nearly zero and thus discordant. These results are reproduced in my Table VII, columns 3, 4, 5, and 6. It should be noted that the change in color-index, inferred from my observed change of type in *l* Carinae from the general spectrum, is closely of the same magnitude as the changes of color-index obtained for the other Cepheids, both photometrically and inferred from the changes in the hydrogen spectrum. Very weak results for changes of type in η Aquilae, from measures of the general spectrum made by Campbell and Wright for other purposes, give an inferred change of color-index of 0.4. Thus my results seem to show practically the same correlation between the color of Cepheids and the general spectrum as was found for the hydrogen spectrum.

The foregoing investigation, begun a dozen years ago, has been interrupted a number of times. Aside from the part of the work completed at the Lick Observatory, portions of the final reductions were made at the Córdoba and University of Michigan observatories. The definitive discussion and preparation of the manuscript for the printer should be accredited to the Department of Meridian Astrometry of the Carnegie Institution of Washington (Albany). It is a great pleasure to acknowledge my indebtedness to these observatories for the time allowed toward this problem.

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EXCITATION STAGES IN OPEN ARC-LIGHT SPECTRA

PART I. SODIUM, POTASSIUM, CALCIUM, STRONTIUM, BARIUM, AND MAGNESIUM

By B. E. MOORE

ABSTRACT

Excitation stages in low-current arc spectra of Na, K, Ca, Sr, Ba, and Mg.—By using high potentials it is possible to lengthen the arc so that the middle region is well separated from the poles, and to reduce the current until few or none of the fundamental spectrum lines of the substance under observation are emitted. Evidently this arc affords a valuable means of studying the relative behavior of spectrum lines. Using currents of from 0.03 to 0.3 ampere from a 2000-volt D.C. dynamo, the author made visual observations of the *variation of spectrum with current*, supplemented by measurements of the intensities of many lines. The results for each of the above-mentioned elements are reported in detail. *Five stages of excitation* were distinguished. Stage I: Lowest-current lines, which extend across the arc; generally resonance lines; including the first member of the principal doublet series of Na and K and of the principal singlet series of Ca, Sr, and Ba. Stage II: Coming in with slightly higher currents; relatively weaker in the middle of the arc; generally ionization lines; including the diffuse and sharp subordinate series of Na, K, Ca, Sr, and Mg and the *p*-series of Ba. Stages III and IV each require a considerably greater current and each has considerably higher ratios of intensity at negative pole to that at positive pole and to middle intensity than the preceding stage. Stage V includes lines which failed to appear at all in the arc. Some bands were also observed coming in at early stages. It was found that the *ionization lines of impurities* such as Na appeared only when the ionization potential of the main element in the arc was greater than that of the impurity. These results will be summarized and discussed in Part II, to follow.

INTRODUCTION

The recent investigations upon low-potential discharge in vacuum[1] have led to some important distinctions which have a bearing upon the formation of spectral lines under all conditions. From these low-voltage experiments there appear to be two early stages of excitation of spectral lines. The first is designated the resonance and the second the ionization stage. The resonance stage contains one or two fundamental lines. The ionization stage contains a *rich* spectrum, which rather inadvertently has been designated the *complete* line spectrum. In these experiments

[1] Tate and Foote, *Philosophical Magazine*, **36**, 64, 1918; *Bureau of Standards, Scientific Papers*, No. 317. Foote and Mohler, *Philosophical Magazine*, **37**, 33, 1919; *Journal of Washington Academy of Sciences*, **8**, 513, 1918; Foote, Rognley, and Mohler, *Physical Review*, **13**, 59, 1919; Mohler, Foote, and Stimson, *Bureau of Standards, Scientific Papers*, No. 368; Davis and Goucher, *Physical Review*, **10**, 101, 1917; and numerous others.

it is further made clear that radiation is produced by recombination of electrons and ions after the force which displaced the electrons has ceased to act. Moreover, the displacing force has its origin in a potential which drives the electrons across the vacuum. These electrons may collide with an atom and will ionize it if the velocity of the electrons is adequate. When one considers the kinetic possibilities it is evident that velocities will be distributed over a wide range of values. But so far as the excitation of a line is concerned, all velocities below a certain limit will be ineffective, and all above the same limit will be effective. This critical velocity will define the beginning of a stage of excitation for that line. As the number of electrons which have a velocity above the critical value increases, the line increases in intensity. In a general way this number of ions will increase with the current. The current may therefore become an index of the development, and may be used instead of the potential and velocity, particularly when the latter are not available.

When one passes from the vacuum to an arc working under atmospheric pressure, one must make one essential distinction. The restraint of the gaseous particles is now such that there can be no free movement over an extended distance. There is also a great difference in the freedom of motion in different parts of the arc. This is due to the great inequalities of temperature in the arc. A large part of the difference in spectra in different parts of the arc have long been attributed to temperature differences and to chemical changes. The new concept does not invalidate the conclusions upon the effect of temperature and chemical action, for doubtless these effects are concurrent. It only shifts the point of view and seeks to analyze the radiation in terms of an electron disturbance produced by an effective potential.

From this standpoint let us take a view of the cross-section of the arc midway between the poles. Where the temperature is lowest in the outer mantle, the electrons have their lowest velocity, and the outside limits of the spectral lines are determined by their critical velocity in that region. Outside this region the electrons may be moving at a lower velocity. Lines which require a higher velocity of the electrons for excitation will not be found in the

outer region. In the hotter interior of this cross-section there is greater kinetic freedom, a more rapidly flowing current, and electrons of higher velocity. Consequently, in passing from the exterior to the interior of the discharge, there is a progressive increase in the velocity of the electrons. Hence one may arrive at a second critical excitation velocity characteristic of other lines upon passing from the exterior toward the interior of the discharge. When this critical velocity is reached, the line or lines to which the critical velocity belongs will show an incipient radiation. There will be an increase in radiation for this line or lines as one progresses farther toward the center of the section. This same argument holds as one passes from the middle along the axis of the discharge until one reaches the poles, near which there is probably a great increase in velocity. Therefore, one might expect to obtain in the pole discharge one or more critical velocities which would produce spectral lines of corresponding stages. The outer mantle, according to this view, would correspond to a low excitation stage, the middle of the arc to a medium excitation stage, and finally the poles would show higher excitation stages. The wide range of velocities in any given position certainly makes the problem more complex. One could go over the literature of arc spectra and make a fair selection of stages for lines with no further distinction than is made here.

The object of the experiments described in this article is to follow the production of lines a little more definitely. To accomplish this, it was necessary to obtain a larger separation of the different stages than is obtained in the usual high-current arc. First, the arc was spread out by deflecting it with a magnet.[1] This was very helpful, but it was soon found that reducing the current and raising the arc potential would isolate the parts of the arc better. The higher-potential arc could be longer, and hence the middle area (rather volume) would be more extended. With the lower currents, the polar illumination retreats toward the pole. With this retreat came another very decisive factor, viz., the disappearance of many lines. Out of this fact arose the determination of stages by determining the current at which the inception of the line occurs. This

[1] Referred to again in Part II (subsequent paper).

method will yield reasonable results unless the line is overlapped by a continuous or band spectrum. But it could not answer one question, viz., Do the lines have definite beginning-points as they appear to, or do they continue in a very weak state to a common excitation point characteristic of all lines? For this reason a method was devised for measuring roughly the development of the intensity of the lines with increase in current. This method shows that certain groups of lines have common beginning-currents, and instead of continuing in a weak state, they frequently disappear and reappear very suddenly and have a very rapid and decisive development after their inception. Knowing this, one could feel perfectly free to use the first course of investigation wherein the lines were followed to their beginning-current. This can be done at either pole or in the middle of the arc or in all of these positions.

THE ARC

As stated above, in the usual arc operated at high current it is difficult to separate the different degrees of excitation. The illumination herein designated "polar" does not appear *on* but just *off* the electrodes. Lines which usually have been designated "short" belong in the polar illumination. They vanish as the current decreases. Many lines which extend across the arc for high currents become short lines for lower currents. This feature helps to distinguish them. These high-potential arcs, according to Simon,[1] may act along two characteristic curves. The first curve is the glow characteristic and belongs to the glow arc. This curve depends only upon the medium and is independent of the electrodes. The second characteristic is the true-arc curve. In the true arc the material of the electrodes affects the electrode drop, and the gaseous products or ions therefrom affect the conduction of the medium. The middle of the low-current arcs under discussion often assumed a glow appearance, but there was at no time any clear evidence that the arc discharge had passed over into the glow form.

[1] Under investigation by Mr. V. L. Chrisler; Simon, *Physikalische Zeitschrift*, **6**, 297, 1905; Simon and Malcolm, *ibid.*, **8**, 471, 1907.

POWER

A dynamo rated at 2000 volts and 0.25 ampere was used. It stands an overload of 20 per cent for brief intervals. The current was adjusted by means of resistance in series.

SPECTROSCOPES

Five spectroscopes have been used. (1) A small student's spectroscope has been used for preliminary investigation. It could be turned with its slit perpendicular or parallel to the discharge. It thus served to observe the variation of intensity in cross-section as well as in length. It was particularly serviceable when the analysis reduced to a few visible lines. (2) A Hilger wave-length spectroscope has given most results. It just separates the D lines, but one could not rely upon its measurements to closer than 6 angstroms. This seldom interfered with identification. The field was usually diaphragmed to about 60 angstroms, which was a great help in determining luminosity. (3) A Steinheil plane-grating spectroscope gave more than double the dispersion of the Hilger spectroscope in the red, and helped particularly in this region. This instrument could be used either visually or photographically. (4) A quartz spectrograph was in good focus from λ 2800 to λ 4000 and gave a spectrum about 18 mm long. When there are but few lines in the ultra-violet, they may be recognized by this instrument.

MEASUREMENT OF INTENSITY

The reasons for following the changes of intensity in the lines as the current changes have been given in the introduction. Two convex lenses with a crossed-nicol system between them focused the light from the arc upon the slit of the spectroscope. Before the eye was placed a Wratten light-filter. After selecting a spectral line for observation a filter was introduced which would nearly extinguish the visibility of the line. The angle between the nicols was then adjusted until only the last trace of the line was visible. The reciprocal of the relative amount of light transmitted by the crossed-nicol filter system is designated the intensity of the

line. This method is open to three large sources of error: (1) personal judgment upon threshold intensities is quite variable; (2) the optical adjustments are subject to some variation; (3) the arc is inclined to migrate. This makes the measurements very rough, but they will serve the purpose of defining the starting-current of spectral lines and of showing their relative development.

DETERMINATION OF STAGES

In an attempt to distinguish stages of excitation the first step is to fix the general features of the spectrum. The last, but more concise, determination will be the growth of the lines themselves.

Stage I.—To recognize this stage, choose a minute current, then look for lines which extend across the whole arc from pole to pole. If these lines are but little stronger at the poles than in the center of the arc, they belong either to Stage I or Stage II. Next see if they can be traced to the last limit of the current. Those lines which hold out longest belong to Stage I. Next examine the rate of intensity development. Lines belonging to Stage I will fairly leap into brilliance at the negative pole and follow but little later at the positive pole. For later currents they will show very slow development. In the middle of the arc they will develop rapidly, but more uniformly. As far as experience with different substances has gone, the current may be reduced until these lines, small diffuse bands, and a low continuous radiation are in complete possession of the spectral radiation.

Stage II.—This stage usually represents a *rich* spectrum. The lines at the negative pole may persist to nearly the same limit as lines of Stage I. When the positive-pole radiation is not too confined, they separate better there than at the negative pole. In the middle of the arc, a line of Stage II must be intrinsically very brilliant to contest with lines of Stage I for first rights. When the intensities of these lines are studied, some of the stronger lines may show a growth very similar to lines of Stage I. But when the current-intensity curves are plotted, they will be seen to lie in a position which indicates stronger exciting currents and stronger inception currents (intercepts upon the current axis) than is required for lines of Stage I. There is furthermore a reluctance

upon the part of these lines to appear in the middle of the arc. This is best shown by the increased ratio of polar to middle intensities.

Stage III.—These lines will not generally show until the inception current of lines of Stage II has been doubled or trebled. When the current is large enough to indicate their decided presence at the negative pole, lines of Stage II will be reaching out far from the poles trying to become true long lines, while lines of Stage III will be cut off sharp and short in a region restricted to the very core of the pole illumination. So far as these experiments have progressed, experience has shown that lines of Stage III require four to five times as much current at the positive pole as at the negative to produce a first visible impression. Within the limits of the present experiments, a line of this stage must be very intense to show any trace in the middle of the arc. Necessarily, therefore, the ratio of their polar to middle intensities is very large.

Stage IV.—It requires a large increase in current over that required for lines of Stage III to bring out the lines of Stage IV. When once they appear, they are very similar to lines of Stage III. There is a decided continuous spectrum associated with most of these lines, which suggests a formation within a dense vapor. None of the lines have been seen outside the pole illumination.

Stage V.—In this class one may place all other known lines of the stronger-current arcs. There has been no attempt to analyze these lines, and, of course, there may be many stages present.

GRAPHS

Logarithms of intensities are plotted as ordinates, and logarithms of one hundred times the current are plotted as abscissae. Logarithms to the base 10 are used. The lines are designated upon the graphs and in the legends.

SODIUM

Lines D_1 and D_2 are predominant. They appear throughout the whole of the arc. With the slit normal to the current they are seen to be longer than the weak continuous background. In this position they fade off at the ends with no defined external

limits. These lines have a very low excitation. The other lines observed were members of the first and second subordinate series and the principal series pair, λ 3303. All of these lines appear in the middle of the arc at 0.25 ampere. With the slit normal to the current they fade off very quickly, but not abruptly, as one proceeds from the center toward the outer mantle. The lines appear to be one-third to one-half the length of D_2. Now Foote and Mohler[1] give an excitation potential for D_2 equal to 2.12 volts, and for the other lines a potential of 5.13 volts. The effective velocity for the latter is 2.4 times the velocity for the former. This means

TABLE I

INTENSITIES FOR D_2

CURRENT	$NaCl_2$		$SrCl_2$		$CaCl_2$	CARBON
	Negative Pole	Middle	Middle	Positive Pole	Middle	Middle
Ampere						
0.25	.....	.....	38	520	8–16	18
.2	7080	2220	24	.....	.....	.....
.15	.....	1280	10	240–180	.....	4.8
.1	2220	532	.....	72	.....	.....
.08	.....	304	.....	.....	.....	.....
.075	1332	.....	.....	28	.....	.....
.067	396	176	.....	.....	.....	.....
.....	(180)	.....	.....	.....	.....	.....
.048	(100)	96	.....	.....	.....	.....

that with a current of 0.25 ampere one must pass from the outer effective excitation of the line D_2 halfway to the center of the current stream to reach a point where the velocity of flow has been doubled. In general, the "fading-off" of lines upon passing from the center to the outer mantle is not sufficiently abrupt to make sharp distinction in stages. As the current decreases, the subordinate-series lines fade out in the center of the arc. The yellow-green members can easily be seen to retreat toward the poles and to shorten. The red and blue members are not visible at the poles below 0.075 ampere, and the yellow-green members not below 0.05 ampere, except during intermittent flaring of the light. The sharp series was not recognized in the preliminary trial

[1] *Loc. cit.*

with the small spectroscope, but was readily found with the Hilger instrument. A cross-sectional view at the poles is interesting. The subordinate series (diffuse and sharp) essentially belong to this region for the lower currents. The D_1 and D_2 lines flash out with great brilliancy and are quite wide. Much of this is doubtless an irradiation effect. With the Rowland grating the D lines show reversal at 0.25 ampere in the pole illumination.

With the crossed-nicol filter system, the data given in Table I were obtained for D_2. Different salts, as represented in the table, were used upon the electrodes, but only this one line was studied.

In passing from the poles toward the middle, the easier lines, particularly lines of Stage I and Stage II, fade off slowly as above noted. This necessitates a little precaution as to what part of the middle one may use to determine middle intensity. Usually the position of weakest intensity is a little above the center of the arc toward the upper pole.

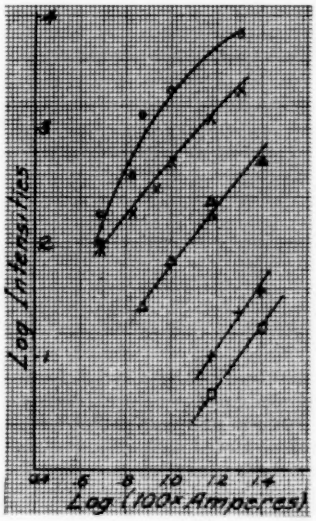

FIG. 1

⊙ $NaCl_2$, −pole
× $NaCl_2$, middle
△ $SrCl_2$, −pole
+ $SrCl_2$, middle
□ $CaCl_2$, middle

A graph, Figure 1, was plotted from the D_2 readings. It will be noticed that the line found in sodium shows two distinct slopes. When the line shows as an impurity in strontium, calcium, and carbon electrodes, its intensity proves to be surprisingly low. With carbon electrodes without salts added, the potential takes a decided increase. This increase in potential actually brings out the D lines stronger in the carbon arc than in the arc containing calcium salt. The slightly greater intensity of the line in the strontium salt suggests the presence of a trace of sodium in the substance.

The D lines belong to Stage I, and the other lines not earlier than to Stage II. The diffuse series certainly fall into this classification.

The decision upon the fainter sharp series was less certain. All other series have not been realized. With substances which follow it is easier to assign the missing lines to later stages. In sodium the writer has more reluctance to do so, because the relative intensity of the continuous background was a burden. With the Rowland grating this background could be dispersed so that it was unimportant, but 0.25 ampere would scarcely produce a trace of the *d* series. Hence there is real need of tracing the growth of the weaker lines at higher currents to determine their excitation stages.

POTASSIUM

The red lines λλ 7699, 7666 (the first term of the principal series of doublets) would naturally be expected to play the same rôle as D_1 and D_2, and the blue lines λλ 4047, 4044 the rôle of the λ 3303 pair, i.e., the former should be a resonance and the latter an ionization pair. Along with the latter would be expected the subordinate series. Although all these types may be seen in the visual spectrum, the red and blue pairs are at the extreme ends, where the sensitivity of the eye is very low. For the author's eyes the sensitivity is much better for the blue lines. But this fact ought not to account for his ability *always* to see the blue lines whenever the red lines are visible. There was no definite answer as to whether or not the blue excitation could be separated from the excitation of the subordinate series. They certainly occur together, but one cannot decide whether they may occur separately, for the subordinate series as well as the blue lines become quite intermittent at even 0.1 ampere. To make any comparison of the intensities, it would be necessary to know the relative sensitivity of the eye for the wide difference in radiations. The crossed-nicol system was not applicable. Evidently the blue pair had a high intensity in comparison to the subordinate lines. Photographically the blue pair are the only lines which show. The latter pair is the second member of the principal series. There are many other lines of conspicuous intensities, but none of them appear under the present excitation currents. This blue doublet λλ 4047, 4044 comes out as an impurity in other substances and with the carbon electrodes.

The D lines show in potassium both as middle and pole lines, but the ionization lines of sodium do not show. According to Foote and Mohler,[1] the ionization potential for potassium is 4.1 volts and for sodium 5.13 volts. It is possible to argue that we have here an effective ionization potential greater than 4.1 volts which produces the potassium lines and less than 5.33 volts which fails to produce the ionization of sodium. The existence of such a narrow range of effective potential seems, prima facie, improbable. Rather, the sodium ionization lines would appear to be absent because of their inherent weak luminosity. However, the following consideration confirms this rather unexpected result. The calcium doublet pair λλ 3969, 3934, the potassium doublet λλ 4047, 4044, the sodium doublet λλ 3303, 3302, and the sodium yellow doublet of the diffuse series, all of which are ionization lines, show prominently in the carbon arc. The ionization potential in this carbon arc is nearly doubled, as is apparent when one considers the potential fall in the different arcs. The ionization value for calcium as given by Foote and Mohler is 6.01 volts. Since the above-mentioned calcium lines appear, we have an effective ionization greater than 6.01 volts. Therefore the sodium ionization lines, which require only 5.13 volts, also appear; whereas sodium did not appear in the potassium arc where the required potential needed to reach only 4.01 volts. The amount of sodium actually present in the two cases could not have been appreciably different.

The red lines which would be expected to belong to Stage I could not be satisfactorily observed. The violet double which is the second member of the same series belongs in Stage II. The same is true of the yellow lines of the diffuse series. The observations could be carried no farther.

CALCIUM

A large number of lines in calcium as well as in strontium and barium were easy to realize. From the intensities which have been observed and the intensities which are recorded in the table for calcium in Kayser's *Handbuch der Spectroscopie* (**5**, 246) it was easy to infer which lines could have been reasonably expected. These lines are given in Table II. The first column indicates the

[1] *Philosophical Magazine*, **37**, 33, 1919.

TABLE II

CALCIUM

	Wave-Length λ	I	Current 0.25 Ampere		Limit of Visibility in Amperes		Stage
			Positive Pole I	Middle I	Positive Pole	Middle	
A	6500	4	18	1—	.....	.....	.....
	6494	4	15	2	0.045	0.12	II
	72	4	8	1—	.07	.....	.....
	63	6R	15	2	.045	.12	II
	49*¶¶	4	5	0	.....	.....	.....
	6439†	10R	20	5	.033	.05	II
B	6170*	6	0	.....	.....	.....	.....
	69	4	0	.....	.....	.....	.....
	67	4	0	.....	.....	.....	.....
	64†	4	8	0	.....	.....	.....
	62‡	10R	15	5	.04	.08	II
	61***	5	.....	.....	.....	.....	.....
	22	10R	8	2	.06	.15	II
	03	8R	8	1	.....	.....	?
C	5868§	6	0	0	.....	.....	V
	58	10	15	—	.035	.....	III
D	5603***	8	.....	.....	.....	.....	.....
	02‖	8	4	0	.045	.....	II
	5599	8	8	1	.045	.....	II
	95	10	10	2	.045	.....	II
	90	8	25	.....	.035	.045	II
	89***	10		5	.....	.....	II
	82¶¶	8	6	0	.....	.....	.....
	5513‖‖	4	.....	.....	.....	.....	III
	5350¶	10	0	0	.1	.3	V
E	5270**	10	8	4	.035	.045	II
	64	8	6	2	.04	.055	II
	5162	6	.....	.....	.....	.....	.....
	60	4	4	2	.04	.055	II
	5042	3	.....	.....	.095	.3	III
F	4878	10	5	.....	.08	.2	III
	47	4	0	.....	.....	.....	.....
	34	1	0	.....	.....	.....	.....
	23	1	0	.....	.....	.....	V
	11.6	.....	0	.....	.....	.....	.....
	07	1	0	.....	.....	.....	.....
G	4685‡‡	4	0	0	.....	.....	V
	25	1	8	3	.....	.....	.....

TABLE II—*Continued*

CALCIUM

	Wave-Length λ	*I*	Current 0.25 Ampere		Limit of Visibility in Amperes		Stage
			Positive Pole *I*	Middle *I*	Positive Pole	Middle	
H	4607§§	.....	20	5	0.035	0.055	I
	4586	30	25	1.+	.038	.1	.....
	82	30	20	1.+	.04	.1	II
	79‡‡	20	10	1	.045	.15	.....
	4527	20	2	0	.12	—	III
I	4455††	200	60	5	.033	.055	.....
	35.1	100	50	5	.033	.055	II
	25.6	100	30	3	.033	.055	.....
	4355	3	2	0	.045	.15	III
J	4319	8	25	1.—	.....	.....	.....
	308	5	20	1.—	.....	.....	.....
	303	9	30	2	.045	.15	III
	299	8	20	1	.....	.....	.....
	90	0	25	1	.....	.....	.....
	83‡‡	8	.....	.....	.....	.....	.....
	4227	1000	200	50	.028	.033	I
	3967	.....	.....	.....	.....	.....	II
	3933	.....	.....	.....	.....	.....	.....

* Bands overlap the group.

† An easy middle line, while the rest of the group are difficult.

‡ First line of the first member of the sharp series. Strong like λ 6439, but not so persistent at low currents. Also an easy middle line, λ 6161, not resolved. Term 3 of this member overlapped by a band.

§ An easy position for observation with very unexpected results.

|| In comparison with other lines equally strong, Group "D" should be stronger in the middle, like λλ 5590, 5589. They are therefore difficult middle lines and may belong to Stage III.

¶ Unexpectedly missing.

** Uncommonly *relatively* strong lines at middle. Easy middle line.

†† Fade off through a distance as large as the pole bead, but are very weak toward the center, and intermittent across the center for low currents. Diffuse-series group, $2p_2-md$.

‡‡ This pair are the inverse of the expected result. Thalen gives four lines, and the one actually present he records as the weakest. Strong background interferes.

§§ A strontium impurity recorded for the sake of comparison.

|| || Lost in band.

¶¶ Exceptionally short line.

*** Unresolved.

anticipated lines and the second column the intensities of the lines taken from Kayser's table. The subsequent columns represent the observations with the Hilger spectroscope. The indexes pertain to some condensed statements in the footnotes in regard to the behavior of the lines. The lines are given in groups "A"

to "J." These are quite arbitrary groups. They represent aggregates of lines, not series grouping. It is largely a convenient method of making comparisons. However, many of these groups behave very similarly and doubtless have much in common. Group "C," for example, belongs to the Ritz combination, $3d_j - 3p_i$[1] There are also certain groups, or aggregates, which fail to appear at all.

Line λ 4227 corresponds to requirements of Stage I. There are numerous lines distinctly belonging to Stage II. Many of these have been examined in a preliminary way with the crossed-nicol filter system. With some lines there is considerable uncertainty; there are many late lines.

With carbon electrodes without calcium chloride the potential takes a big jump. The three leading lines which show in the pole illumination at the carbon are then λ 3934, λ 3968, and λ 4227, and in the order named. At 0.1 ampere, electrode λ 3934 is about three times as bright as λ 4227. These three lines occur as an impurity in barium, but a good share of the calcium there doubtless comes out of the electrodes. Again we find at the poles with 0.1 ampere an intensity about 50 for λ 3934 and about 20 for λ 4227, while in the center λ 3934 does not show and λ 4227 has an intensity of about 8. It will be noticed here that there is both a higher temperature and a larger potential fall at the poles; and that both of these factors will contribute to the production of higher-velocity electrons. This is the essential requirement of Stage II, while the middle of the arc for the lowest currents corresponds rather closely to Stage I. Therefore, with the increased velocity of the electrons, the lines λ 3969 and λ 3934 of Stage II replace the lower-excitation line λ 4227 of Stage I. If this consideration has merit, it is suggestive as to solar conditions, where these lines are of predominant interest, and where the doublet pair are not only the brightest, but also the most conspicuous lines in the entire solar spectrum.

From the results of Mohler, Foote, and Stimson,[2] it is shown that λ 6573 is the first and λ 4227 the second resonance line in calcium. It appears that resonance has been observed in three

[1] Popow, *Annalen der Physik*, **45**, 154, 1914.

[2] *Bureau of Standards*, Paper No. 368.

types of series, $1S-mP$, $1s-mp_2$, and a cross of these systems, $1S-mp_2$. Lines λ 3934 and λ 3968 represent the second type, clearly pole lines in the arc, and λ 4227 represent the first type, which is an extremely easy line in the middle of the arc. Line λ 6573 has always been unimportant in the arc. It represents the cross-system. King[1] finds it brilliant in the furnace spectrum of calcium. It seems therefore to deserve close attention. In the present arc the conditions almost baffle investigation. The line at the pole is doubtful even at 0.25 ampere. In the middle it intermittently comes out of a halo or narrow band, when it is brighter than its near neighbor λ 6500. (See λ 4571, magnesium.) Lines λ 5042, type $1D-2P$, and λ 4527, Type $1D-3P$, are lines which are just traceable under the experimental conditions in the middle of the arc at 0.3 ampere. They clearly belong to Stage III. Careful search for λ 5513, Type $1P-3S$, and for λ 4847, Type $1P-4S$, failed to discover them. Hence they must be assigned to a later stage.

The calcium spectrum nearly fades out to the blue of λ 3934. The lines of the second member of the sharp series of triplets are near neighbors of λ 3934. These lines are λλ 3974, 3957, and 3949. The first two lines have about 1/200 of the intensity of λ 3934, and the third line is invisible. Lines λ 3737 and λ 3734 have the same frequency differences as λ 3939 and λ 3934. The intensities for these two lines are respectively about 1/50 and 1/200 of λ 3934. These five lines belong essentially to Stage III, while the diffuse-series' first and second members are found in Stage II. The head of the so-called cyanogen band at λ 3883 is traceable.

The details of the crossed-nicol measurements are very similar to those found with barium lines and are not given here. The readings for a few prominent lines at 0.25 ampere only will be given. It is interesting to compare these values with the estimate of the unaided eye in Table II. The lines are given in angstrom

[1] *Astrophysical Journal*, **28**, 389, 1908. King gives the literature concerning these three lines and an examination of them in the electric furnace with different quantities of calcium and at different temperatures. Our results are very similar. He attributes the growth of λ 3969 and λ 3934 to increase in temperature. In the present discussion temperature is recognized as a contributing cause and the participating cause is held to be the state of excitation.

units, with the measured intensities in parentheses following the lines: λ 5589 (30–50), λ 5485 (370), λ 5270 (133), λ 4878 (133), λ 4586 (40), D_2 (21), λ 5934 (370).

BANDS

Line λ 5485 is approximately the blue edge of a band, which contracts in width as the crossed angle of the nicol system increases. Another band, λ 5934, adjoining D_2 (sodium), seems more like a line in the midst of a broad radiation. As the crossed-nicol angle increases, the band contracts toward the line, but fades out more rapidly upon the less refrangible side. Judgment upon the lines was made at the poles and upon the bands in the middle of the spectrum. Band λ 5934 is much stronger at the pole than in the middle. It is, however, very fickle at the pole and some readings showed twice the intensity of others. Hence they were discarded. Band λ 5485 is different. It is weaker at the poles; its appearance, which was observed with a 20 per cent crossed-nicol, is shown in Figure 2. Adjoining the band on the red is Group *d*, of pole lines. In the pole region the band fades off upon its weaker side. It is strongest just above the limit of the pole lines. Another line or narrow band, about λ 6080 to the blue of λ 6103, behaves in the same manner as λ 5480. It requires no nicol to observe it. Opposite the pole line λ 6103 it is very weak. Line λ 6103 shows weakly as a middle line, and opposite its middle λ 6080 is much stronger than opposite the pole. Some bands of the chloride group are given in Kayser's tables in the region of λ 5480, and λ 5934 is there assigned to the metallic group. This suggests that the chloride molecule does not exist in the position of the pole excitation, and that there is a large recombination of metal and chlorine after passing beyond the pole excitation into the region of lower excitation. The metallic (molecular) excitations are just as persistent in the middle of the arc as the leading lines. They can be traced to the last limit with λ 4227, the resonance vibrator. Unfortunately the writer could not use the crossed-nicol with the latter line. Its behavior, in general, is like D_2. The band λ 5934

FIG. 2

grows in intensity with the current at the middle, not in two well-defined branches like resonance lines, but more like other lines, with moderate and progressive changes in the slope.

Some lines have been added to Table II since the receipt of an important contribution by Dr. F. A. Saunders.[1] His article gives one enough types to make some comparisons, which one may safely make and reserve the discussion of the matter to the subsequent contribution. The line of Stage I has Type $1S-1P$, while Type $1S-1p_2$ is missing. When ionization occurs and Stage II appears, Types $1p-1s$, $1p_i-2d$, $1p_i-3d$, and $1p_i-2s$ appear. Stage III is represented by Types $1P-3D$, $1D-2P$, $1D-3D$, $1D-1F$, $1D-2F$. Observe here an expansion of the atomic action as represented by the first term of the types and of the electron displacement as represented by the second term of the types in passing from Stage I to Stage III. A spectroscopic observation may also be added. Lines of Stage III come out of the very core of the polar illumination where there is a great vapor density, strong, continuous spectrum, greatest current density, and a *sustained* action upon the ions.

STRONTIUM

There will be no particular advantage in tabulating the observed lines as in calcium. Using Kayser's *Handbuch der Spectroscopie*, **6,** 539, as a guide, all strong lines given by Exner and Haschek for the arc are obtainable in the visible spectrum. Their lines of intensities 1 and 2 only are uniformly missing. Besides representatives of this weaker class of lines, there are seven lines missing between λ 5971 and λ 5599 inclusive.

These missing lines are not appreciably disturbed by continuous or band radiation, they therefore belong to late excitation stages. Lines λ 4338, λ 4319, and λ 4308, a triplet group, are overlapped by band or continuous radiation, and only the first line of the group is visible. These lines at the poles show no conspicuous variation from the values of relative intensity given by Exner and Haschek. The stronger the pole line, the more easily it is in general to trace it toward the middle of the arc. But

[1] *Astrophysical Journal*, **52**, 265, 1920.

strong pole lines do not always come out with the same ease at the middle. Some lines relatively weaker tend to come out at the middle much sooner than stronger lines. Because of this characteristic, it seemed desirable to tabulate the lines as in calcium. But there is too much diversity in this variation to reduce it to any system. In trying to trace the lines to the lowest current limit, the weaker lines go out first, invariably, in strontium. There are a number which vanish about 0.045–0.05 ampere at the negative pole. But many of these lines are still distinct at the negative pole at 0.032 ampere. Just off the center of the negative glow, either toward the pole or toward the center, there are only three lines, λ 4607, λ 4216, and λ 4078. On the outer mantle of the negative pole, and in the center much of the time, λ 4607 only is found. At the positive pole there are only the three lines at 0.05–0.055 ampere. Here, there is the first trace of a richer spectrum at about 0.07–0.08 ampere. It requires a current of 0.1 ampere at the positive pole to produce a many-lined spectrum as rich and strong as is found at the negative pole at 0.035 ampere. Later, by using 3000 volts, the current was carried down to 0.025 ampere. The three lines listed above are still visible at the negative pole, but the "*p*" pair of lines is very weak. At the positive pole and in the middle of the arc, λ 4607 alone is visible. Evidently the positive pole separates the excitations better. There is now only a low, wavelike illumination in the red-band region.

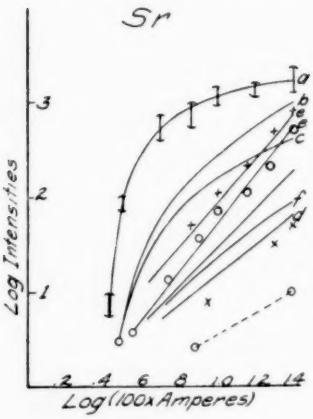

FIG. 3

a=λ 4607, −pole
b=λ 4607, middle
c=λ 4962, −pole
d=λ 4962, middle
e=λ 6025, band−pole
f=λ 6025, middle
+, λ 4873, −pole
×, λ 4873, middle
○, λ 5950, pole

A few of the lines have been studied with the crossed-nicol and filter combination. They are given in Figure 3. Line λ 4607 shows two distinct slopes. In the second slope, it was quite

impossible to secure readings which did not overlap. The slope is here less steep than for the other substances. The prime cause lies in the strong self-reversal of the line. It was found that reversal was evident at 0.1 ampere and became more pronounced as the current increased. This is the most conspicuous line in the strontium spectrum. Its series type is $1S-mP$, and the leading term of this series. Line λ 4227, in calcium, has been identified with this type, but it could not be followed. Lines D_1 and D_2 in sodium show the same characteristics as this line, but are represented by the series Type $1s-mp_2$. In strontium the latter type is represented by λ 4126 and λ 4078, as is clearly proved by the Zeeman effect. These cannot be followed satisfactorily in a quantitative way. It is clear from photographs with low currents that λ 4607 is predominant, and for larger currents that the lines λ 4216 and λ 4078 are overtaking the former. But even at 0.25 ampere it could not be said that they intrinsically equal λ 4607. The latter is a very wide line with strong self-reversal. As a middle-arc line, λ 4607 has no lower slope at 0.25 ampere than the same line at the pole with 0.05 ampere. The change in the line at the middle of the arc is more progressive. Even at the middle of the arc at low current it is more prominent than any polar line. Up to 0.25 ampere, no polar line, unless it be λ 5481, has equaled the intensity of this middle line.

Line λ 4962 is a second member representative of the diffuse series, while the first member lies in the extreme infra-red, at λ 2735–6, and cannot be studied. Line λ 4962 is a relatively easy middle line. Its characteristics show a development more like λ 4607 than had been expected. The development is slower. As a pole line it follows λ 4607 as a middle line for a short distance and then progressively falls off in intensity more than that line. Now too much stress may be laid upon these appearances. But realizing the apparently easy interpretation which could be placed upon it, the readings for the line were repeated three times by varying nicol at constant current, and by varying the current with constant nicol. This gave a *promiscuous lot* of readings along the line, but did not change the feature of a rapid early, and slow later, growth in intensity with increasing current.

Line λ 4968 is the stronger of the two satellites of λ 4962. It increases both at the pole and at the middle very uniformly. It is so very weak at the middle that there appears a great tint difference between it and its near companion λ 4962. At 0.25 ampere it is noted that the intensity of λ 4962 at the pole is twenty times as strong as at the middle, whereas λ 4962 pole is fifty times as strong as λ 4968 middle, i.e., we have here intensity ratios 50, 20, 1. The unobserved satellite λ 4972, according to Exner and Haschek, has an intensity of 3, and for the three lines they give ratios for intensity of 50, 20, and 3. Considerable allowance must be made in the methods of determining intensities. This is scarcely adequate to cover the difference observable. For, easily, 1/150 of the stronger line would have been observed in the clear field at λ 4972. This is nine times the ratio of values required from the Exner and Haschek values. It seems rather reasonable to conclude, therefore, that λ 4972 has not been excited at all. The same method of examination may be applied to missing lines whose intensity is about two, but it does not have quite as much force. Prolonged photographic exposure also failed to bring out these lines. It therefore seems probable that these lines are not excited in the low-current arc. The lines of the second subordinate or sharp triplet series are much less pronounced than those of the diffuse series. The first member lies in a band region, but lines can be recognized at the pole. There is a trace only of two lines, λ 4438 and λ 4362, of the second member. This is out of all proportion to the corresponding member of the diffuse series. For the two respective members Kayser's *Handbuch* gives very similar intensities. Within the first member of the sharp series lies λ 6892, which, the author strongly suspects from King's[1] results, belongs to $1S - m_2$,[2] mixed-series type, just as the calcium line λ 6573. When the pole illumination is accurately upon the slit, there is sufficient intensity here for visual observation. The diffuseness is even more pronounced than for λ 6573, and although having looked diligently, it is not possible to say there is a line in this diffuseness.

[1] *Astrophysical Journal*, **51**, 184, 1920.

[2] Saunders confirms this classification.

The two red bands studied are doubtless each overlapping bands. The wave-lengths assigned represent approximately positions of maximum intensity. Band λ 6025 is more than twice as wide as λ 5950. Both at the pole and the middle these bands increase uniformly with the current.

As seen from the graph, Figure 3, pole line λ 5481, band λ 6025, and band λ 5950 are remarkably alike in their development and growth. After they are definitely established they grow uniformly with the current. They can be expressed by the equation[1]

$$\log I = k\,(\log i - \log i_0), \qquad I = \left(\frac{i}{i_0}\right)^k,$$

where I is the intensity, t the current, i_0 and k constants. Now i_0, the intercept upon the current axis, would be the inception current provided we may extrapolate along the curve below the lowest readings. After inception the intensity is proportioned to some power of the current. The apparent effect is that lines go out a little more rapidly down in this low-current region, but the whole effect is then masked by the hiding of the pole behind some small particle of salt or electrode material. Taking this feature into consideration, the graphs suggest a common origin for all lines studied except for λ 4607. The violet p series λ 4216 and λ 4078 could not be studied by the crossed-nicol method.

Basing conclusions upon the foregoing analysis, λ 4607 is the last limiting line upon reducing the current and belongs to Stage I. It is believed there is clear evidence at the positive pole that the "p" pair does not hold out as long as λ 4607; but whether or not its appearance slightly earlier than the many-lined spectrum was due to its intrinsic brightness could not be settled. If we make a separate stage for this pair, then the many-lined group would fall in Stage III. But these lines correspond very closely to the requirements of Stage II and are like the lines of this stage in other substances. Forty-six lines were counted in Stage II off the positive pole at 0.1 ampere. Lines of Stage III do not isolate well, as the analysis shows. Following the outline of stages, the unobserved lines would fall in Stage V, the unclassified stage.

[1] See Part II (subsequent paper).

BARIUM

The barium lines do not in general push as rapidly toward the middle of the arc as calcium or strontium. Nearly every line of intensity 3 or greater given in Kayser's *Handbuch* (**5**) is present in the pole illumination at 0.3 ampere. The following lines were missed (the parentheses show intensities by Hoeller): λ 6451 (6), λ 6175 (3), λ 5908 (4), λ 5819 (3), λ 5784 (3), λ 5714 (3), λ 5680 (5), λ 5594 (3), λ 4283 (8). The ultra-violet shuts off very rapidly. Line λ 3892, Stage II, is the most pronounced. Evidence for its similarity to λ 5536 was sought but not found. It is followed by a triplet of Stage II, λ 3545, λ 3525, λ 3501, the last of which is the strongest. These lines are not a true triplet according to Saunders.[1] The last line he classifies as the first member of the singlet series, $1S-mF$. The second member of Saunders' singlet series, $1S-mP$, of which λ 5536 is the leading member and the fundamental of the whole spectrum, is λ 3072. Air bands interfere in its observation. The Zeeman effect postulated for this series is the normal triple "*a*." The writer's observations[2] confirm this result for λ 5536, but show a value (12/11) *a* for the second member. The latter is a difficult line, and an error of 1/11 is not excluded. Members 2 and 3 of the *d* series are in evidence at the poles in the ultra-violet. Stage III seems to fit them just as it does the first member (see below). The first term of the first member of the *d* series is missing in barium, just as it was missing in calcium and strontium. Therefore it appears that the satellite types may arise as later excitations than the lines with which they are usually associated.

Increasing the potential in later experiments to 3000 volts, the current was carried down to 0.022 ampere by shortening the arc a little. The pole excitation may then conceal itself between the salt particles, or it may seize upon a salt particle and stand out prominently. In the latter case the spectrum is a beautiful green glow. Bands and the last trace of λ 4554 disappear. Line λ 5536 stands out definitely in the middle of this green spectrum. (Chloride bands are said to overlap this point.) This spectrum, growing

[1] *Astrophysical Journal*, **51**, 23, 1920.

[2] *Ibid.*, **28**, 1, 1908.

continuously weaker, extends far toward λ 4934. Toward the red it extends as far as, or farther than, the position of the D line, but the D lines are absent. In the red there is a succession of weak ripples—presumably a residue of the red bands.

In barium, some lines of the prominent types of series are very favorably located for study by the crossed-nicol filter combination. The lines also lie very advantageously with respect to each other for comparison. The leading singlet member λ 5536 lies close to λ 5519, the strongest line of the first member of the diffuse series. Now if λ 5519 were alone, its sudden appearance with change in current could be attributed to a drift to and fro of the image upon the slit of the spectroscope. But with λ 5536 pole illumination strongly in the field of view at the time of the current change, a sudden appearance has its origin in an outburst of the line. Now these lines are shown in the graph, Figure 4. It appears that line λ 5519 at its inception leaps very quickly to quite a distinct intensity. But it appears later than lines of Stage II, and reluctantly appears at the middle. It is therefore assigned to Stage III, which is a rather unexpected thing for a leading member of the *d* series. Likewise λ 4934 of the *p* series and λ 4900 of the *s* series are adjacent. Line λ 4900 makes a late appearance and is a rigid pole line. The same is true of its companion line λ 4525 which cannot be separated from λ 4523 for the crossed-nicol study. Both these lines are short lines. These series lines cannot therefore get into Stage III, much less get into Stage II, where there is the first appearance of the many-lined spectrum. The earliest assignable stage for this *s* series pair is therefore Stage IV. The pair λ 4934, λ 4554, *p* series, are contenders with λ 5536 for first place.

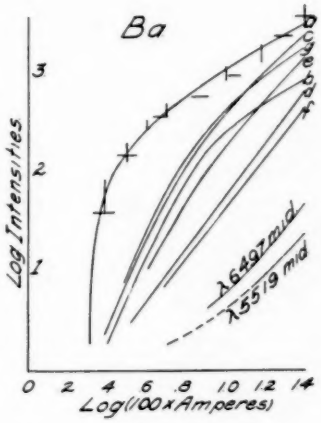

FIG. 4

a = λ 5536, −pole
b = λ 5536, middle
c = λ 4554, −pole
d = λ 4554, middle
e = λ 4934, −pole
f = λ 4934, middle
g = λ 5519, −pole

Following the current down to its lowest limits, they drop out. Along the graphs they have another characteristic. They are therefore assigned to Stage II. Another much weaker pair, λ 6497 and λ 5853, are assignable to Stage II. Line λ 6142 also belongs in this stage. It is generally recognized as a leading line of the barium arc.[1] From Table III it is the sixth line in brightness,

TABLE III

Wave-Length λ	Intensities at 0.2 Ampere		Stage
	Negative Pole	Middle	
6497	276	40	II
6142	268	14	II
D_2, Na	.....	530	I
5853	364	24	II
5778	1150	.....	III
5536	2500	(560)	I
5519	1460	(—)	III
4934	1080	250	II
4900	220	0	IV
4727	660	0	IV
4700	104	0	IV
4691	660	0	IV
4628	355	0	IV
4620	436	—	IV
4580	272	—	IV
4574	288	—	IV
4554	1720	316	II

instead of the fifth,[1] in the middle of the arc; and at the poles it stands much lower than several pole lines of Stage IV. In Figure 4, graph "*f*" represents equally well λ 4934 middle, λ 4900 pole, and λ 6497 pole.

As designated above, there were numerous other lines which are not recorded in Table III. If any of these belong to Stage II, they have escaped the writer's notice. There has been no attempt to separate them into Stages III and IV, where they all belong. Certain unobserved prominent lines noted above may be provisionally placed in a later stage.

MAGNESIUM

First, magnesium chloride was tried upon carbon in the same manner as with the other salts. The salt rapidly disappeared and

[1] Kayser, *Handbuch der Spectroscopie*, **5**, 144–146.

the potential climbed toward the carbon electrode characteristic. For low currents it then assumed the distinct glow arc characteristics. The first member of the sharp series occurred at the negative pole in the glow discharge. Whether or not this member may continue to the lowest limits of the current is uncertain. Much seemed to depend upon the presence of the salt and whether it was participating in the action of the arc. The same holds with reference to the first term of the diffuse series in the ultra-violet, but this was more difficult because photography had to be applied. Neither set of lines appear in the middle of the arc below 0.07 ampere, and probably not below 0.09 ampere. Three leading lines of the oxide group beginning at λ 5007 can be seen for at least as low a current as 0.09 ampere. There are other groups close to the electrode surface also in the ultra-violet which were not identified. Particular interest centers in the line λ 4571. It is the fundamental resonance line of the substance,[1] $1S-2p_2$, mixed-series type. This type was not observed in strontium and very doubtfully observed in calcium. The line was not observed for any current either in the middle or at the poles with the chloride of magnesium. The metal magnesium was next tried. A small piece of the metal was used in a shallow crater of copper, and copper was used for the upper electrode. With a current of 0.07 ampere or less one could sustain the current without flaming for a short time and make a good inspection. The oxide group λ 5007 could be easily traced, and λ 5183 flares, but this line did not show until flaming ensued. This was not a flaming arc, but a combustion which would continue after the arc circuit was broken. The line may be observed in the arc for very large currents, if the salt is plentifully supplied. It then occurs in the middle of the arc, and after watching it some time, it was decided that it occurred by reason of a certain amount of *spraying* of the salt by the hot arc. The same thing could be happening in the larger-current calcium and strontium arcs. It is suggested that this is the solution of the question and that the type of line is foreign to the operation of these arcs. Another resonance line occurs at λ 2852. This line was

[1] *Loc. cit.*

found in the chloride spectrum, but it was not found without the presence of the *d* member λ 3838 and three or four other lines, not well separated, at λλ 2803–2790. The stage of these lines was not settled. They show well at 0.12 ampere. Only the first members of the *d* and *s* series could be safely classified, and then reliance must be made largely upon the ratio of polar to middle intensities. They are assigned to Stage II. Two other polar lines are found which were observed at both poles. These lines λ 5528 and λ 4703 are not earlier than the third stage.

Discussions and the conclusions will follow in Part II, which will deal with metals.

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UNIVERSITY OF NEBRASKA
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A PHOTOMETRIC STUDY OF Y CAMELOPARDALIS

By RAYMOND SMITH DUGAN

ABSTRACT

Eclipsing variable Y Camelopardalis.—The 1035 *photometric observations*, in which B.D.+76°.285 was used as comparison star are given in a table of 210 normals. The probable error of a single observation is $\pm 0^{M}.044$. The *light-elements* derived from the observations are in good agreement with Miss Harwood's conclusions concerning variation of the period. The *light-curve* shows the effects of ellipticity and reflection. A secondary minimum has been detected. The eclipses are partial and last about $12\frac{1}{2}$ hours each. The range of variation is $1^{M}.73$ in primary minimum and $0^{M}.05$ in secondary minimum. "Uniform" and "darkened" *orbital elements* are collected at the end of the paper. In order to get a satisfactory "darkened" solution, it was necessary to assume that one side of the fainter star was entirely dark.

The variability of Y Camelopardalis (B.D.+76°.286) was discovered by Mme Ceraski in 1903. According to Miss Cannon, the spectrum is between A and F. The faintness of the star—Wendell[1] makes it $10^{M}.6$ at maximum—makes impossible a precise classification. The star has been observed—with the usual concentration on the primary minimum portion of the light-curve—by Blajko,[2] Nijland,[3] Wendell,[4] and Lehnert.[5] Miss Harwood[6] has utilized the Harvard plates to form a photographic mean curve and has studied the variation of the period. Mean curves have been discussed and orbital elements derived by Dr.[7] and Mrs.[8] Shapley. My own observations were begun in 1906, but the bulk of the 16,560 measures were made in 1908–1909 and 1916–1917. The observations and discussion will appear in detail in *Contributions from the Princeton University Observatory*.

With the addition of the visual observations the plot of the difference between the time predicted with Miss Harwood's

[1] Wendell, *Harvard Annals*, **69**, Part II.

[2] *Astronomische Nachrichten*, **163**, 91, 1903; **177**, 119, 1908.

[3] *Astronomische Nachrichten*, **176**, 165, 1907; **183**, 281, 1910.

[4] *Harvard Annals*, **69**, Part II.

[5] *Astronomische Nachrichten*, **200**, 163, 1915.

[6] *Harvard Annals*, **84**, No. 2.

[7] H. Shapley, *Contributions from the Princeton University Observatory*, No. 3.

[8] M. B. Shapley, *Astrophysical Journal*, **46**, 56, 1917.

constant period and the observed time retains the general character of her diagram. The residual which apparently increased to a maximum of $+50^m$ in J.D. 2416300 decreased steadily to -2^h24^m in J.D. 2421300. My own observations are best represented by a straight line which makes a slight angle with the contemporary portion of Miss Harwood's curve, and which corresponds to the elements:

$$\text{Min.} = \text{J.D. } 2418306.23526 + 3.3054517\,E$$

The mean epoch of these elements is J.D. 2418521. B.D.$+76^\circ.285$ was used as comparison star and gave no indications of being a

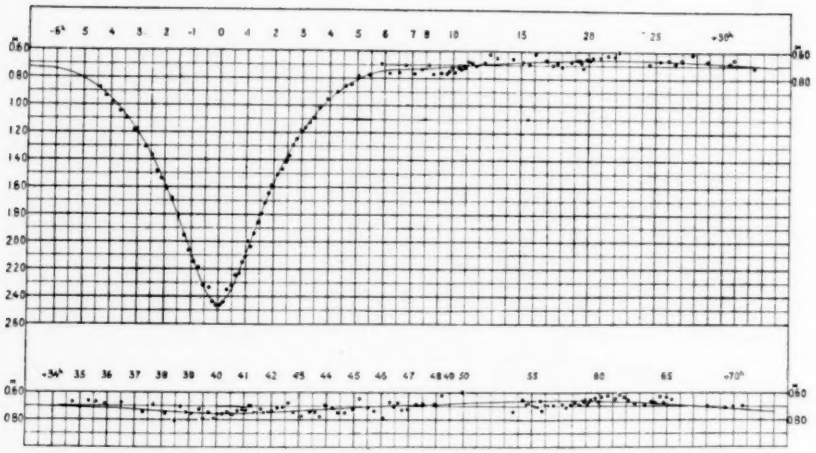


FIG. 1.—Y Camelopardalis

variable. A probable error of $\pm 0^M.044$ of a single observation (16 measures) is indicated by the residuals of the normals of observation from the theoretical curve.

The curve of primary minimum is sensibly symmetrical, results from a partial eclipse, is about $12\frac{1}{2}$ hours in length, and involves a loss of $1^M.73$. A secondary minimum of the same duration and involving a loss of but $0^M.05$ has been found. An apparent displacement of secondary minimum nearly an hour toward the following primary minimum is the only indication of orbital eccentricity.

TABLE I

TABLE OF NORMALS

Normal Number	Phase	Weight	$v-b$		O−C
			O	C	
1	$-0^d4^h24^m20$	4	$+0^M872$	$+0^M883$	-0^M011
2	4 11.90	4	0.936	0.919	+ 17
3	3 56.88	3	0.978	0.962	+ 16
4	39.50	4	1.049	1.022	+ 27
5	25.19	3	1.098	1.083	+ 15
6	3 8.28	3	1.183	1.158	+ 25
7	2 42.94	3	1.306	1.287	+ 19
8	28.00	5	1.366	1.372	− 6
9	16.40	5	1.485	1.450	+ 35
10	2 6.80	5	1.536	1.520	+ 16
11	1 56.33	4	1.613	1.599	+ 14
12	43.50	4	1.682	1.695	− 13
13	29.56	5	1.803	1.820	− 17
14	16.78	4	1.953	1.935	+ 18
15	1 5.94	4	2.064	2.039	+ 25
16	0 54.50	4	2.148	2.131	+ 17
17	44.56	4	2.189	2.225	− 36
18	33.50	4	2.318	2.318	± 0
19	20.56	4	2.335	2.399	− 64
20	12.17	5	2.438	2.430	+ 8
21	−0 0 4.39	5	2.464	2.451	+ 13
22	+0 0 3.94	4	2.463	2.452	+ 11
23	11.10	5	2.440	2.432	+ 8
24	19.10	5	2.350	2.402	− 52
25	29.06	5	2.319	2.347	− 28
26	36.67	5	2.244	2.290	− 46
27	45.80	5	2.230	2.215	+ 15
28	52.63	4	2.150	2.151	− 1
29	0 58.50	4	2.109	2.100	+ 9
30	1 3.38	4	1.998	2.056	− 58
31	10.28	5	2.038	2.000	+ 38
32	17.50	5	1.940	1.931	+ 9
33	26.00	5	1.860	1.850	+ 10
34	34.00	5	1.792	1.778	+ 14
35	41.20	5	1.713	1.715	− 2
36	48.22	4	1.644	1.658	− 14
37	1 55.50	4	1.586	1.609	− 23
38	2 7.33	4	1.507	1.510	− 5
39	18.11	5	1.469	1.437	+ 32
40	25.50	5	1.417	1.390	+ 27
41	33.11	4	1.371	1.340	+ 31
42	40.67	4	1.294	1.299	− 5
43	48.40	5	1.249	1.252	− 3
44	2 58.10	5	1.182	1.206	− 24
45	+0 3 7.13	4	+1.163	+1.163	±0.000

TABLE I—*Continued*

Normal Number	Phase	Weight	$v-b$		O−C
			O	C	
46	+0d 3h 16m 56	4	+1M127	+1M120	+0M007
47	25.29	3	1.091	1.081	+ 10
48	37.56	4	1.021	1.030	− 9
49	3 54.70	4	0.953	0.969	− 16
50	4 16.80	4	0.900	0.902	− 2
51	34.10	4	0.857	0.860	− 3
52	4 47.50	4	0.846	0.832	+ 14
53	5 2.20	4	0.782	0.804	− 22
54	26.70	4	0.773	0.769	+ 4
55	5 53.00	4	0.695	0.748	− 53
56	6 11.43	2	0.762	0.738	+ 24
57	32.50	4	0.758	0.735	+ 23
58	6 48.40	4	0.703	0.733	− 30
59	7 10.70	4	0.764	0.730	+ 34
60	7 41.60	3	0.733	0.728	+ 5
61	8 8.50	3	0.701	0.726	− 25
62	8 33.30	3	0.768	0.722	+ 46
63	9 8.70	3	0.760	0.719	+ 41
64	31.20	5	0.766	0.717	+ 49
65	42.20	5	0.747	0.714	+ 33
66	9 57.80	5	0.724	0.713	+ 11
67	10 9.90	5	0.754	0.711	+ 43
68	24.30	5	0.723	0.710	+ 13
69	36.90	4	0.734	0.708	+ 26
70	10 51.50	4	0.720	0.707	+ 13
71	11 9.40	5	0.678	0.704	− 26
72	24.50	4	0.687	0.702	− 15
73	11 43.00	4	0.709	0.700	+ 9
74	12 19.00	3	0.684	0.696	− 12
75	12 42.10	4	0.622	0.693	− 71
76	13 13.30	3	0.648	0.690	− 42
77	14 23.90	3	0.654	0.683	− 29
78	15 7.20	3	0.678	0.680	− 2
79	15 36.00	4	0.695	0.678	+ 17
80	16 4.30	3	0.618	0.674	− 56
81	16 51.90	3	0.662	0.671	− 9
82	17 22.40	4	0.700	0.670	+ 30
83	17 41.90	4	0.687	0.669	+ 18
84	18 2.90	3	0.711	0.667	+ 44
85	18 42.70	3	0.675	0.666	+ 9
86	19 9.70	4	0.660	0.664	− 4
87	18.90	5	0.663	0.664	− 1
88	28.50	5	0.680	0.663	+ 17
89	36.80	4	0.720	0.663	+ 57
90	+0 19 51.30	4	+0.653	+0.662	−0.009

TABLE I—*Continued*

Normal Number	Phase	Weight	*v–b*		O–C
			O	C	
91	+0d20h 4m60	4	+0M663	+0M661	+0M002
92	23.90	3	0.642	0.661	− 19
93	20 53.20	3	0.640	0.660	− 20
94	21 32.90	3	0.628	0.660	− 32
95	21 57.31	3	0.632	0.660	− 28
96	0 22 15.10	2	0.600	0.660	− 60
97	1 0 31.00	3	0.688	0.661	+ 27
98	1 18.20	3	0.666	0.664	+ 2
99	1 58.25	3	0.659	0.666	− 7
100	2 26.57	2	0.684	0.668	+ 16
101	2 55.22	3	0.656	0.670	− 14
102	3 42.80	3	0.616	0.672	− 56
103	4 50.25	1	0.665	0.677	− 12
104	6 30.69	3	0.680	0.687	− 7
105	6 48.00	2	0.653	0.689	− 36
106	8 18.63	1	0.713	0.697	+ 16
107	9 21.33	1	0.698	0.701	− 3
108	10 43.50	3	0.672	0.710	− 38
109	11 19.10	4	0.664	0.712	− 48
110	35.90	4	0.672	0.713	− 41
111	11 55.00	4	0.686	0.716	− 30
112	12 31.61	4	0.675	0.719	− 44
113	13 18.71	2	0.746	0.729	+ 17
114	13 41.11	3	0.693	0.734	− 41
115	14 7.20	3	0.754	0.738	+ 16
116	28.70	4	0.804	0.742	+ 62
117	41.10	4	0.702	0.745	− 43
118	14 52.90	4	0.746	0.747	− 1
119	15 4.50	4	0.766	0.750	+ 16
120	21.81	3	0.725	0.752	− 27
121	30.22	4	0.792	0.753	+ 39
122	42.44	5	0.746	0.755	− 9
123	15 54.00	4	0.791	0.756	+ 35
124	16 2.20	5	0.759	0.756	+ 3
125	12.60	4	0.758	0.756	+ 2
126	23.50	4	0.741	0.756	− 15
127	34.30	4	0.763	0.756	+ 7
128	45.40	4	0.735	0.756	− 21
129	16 56.10	5	0.729	0.755	− 26
130	17 4.00	5	0.730	0.755	− 25
131	10.90	5	0.691	0.754	− 63
132	16.50	5	0.697	0.753	− 56
133	22.20	4	0.746	0.752	− 6
134	29.70	5	0.747	0.752	− 5
135	+1 17 36.70	5	+0.721	+0.750	−0.029

TABLE I—*Continued*

Normal Number	Phase	Weight	v−b		O−C
			O	C	
136	+1d 17h44m20	5	+0M747	+0M749	−0M002
137	17 52.00	5	0.741	0.748	− 7
138	18 0.20	4	0.734	0.747	− 13
139	12.60	4	0.713	0.744	− 31
140	24.20	4	0.708	0.741	− 33
141	36.60	4	0.678	0.740	− 62
142	18 47.60	5	0.744	0.738	+ 6
143	19 5.80	4	0.774	0.732	+ 42
144	24.00	4	0.743	0.729	+ 14
145	35.80	5	0.746	0.727	+ 19
146	46.00	5	0.778	0.725	+ 53
147	19 56.40	4	0.694	0.722	− 28
148	20 13.20	4	0.710	0.720	− 10
149	25.20	4	0.748	0.717	+ 31
150	40.60	4	0.748	0.713	+ 35
151	1 20 58.50	4	0.724	0.710	+ 14
152	21 12.39	3	0.647	0.707	− 60
153	30.06	3	0.708	0.703	+ 5
154	21 44.30	3	0.736	0.702	+ 34
155	22 3.94	3	0.786	0.700	+ 86
156	18.39	4	0.674	0.697	− 23
157	27.44	4	0.703	0.696	+ 7
158	36.50	3	0.676	0.695	− 19
159	45.60	4	0.729	0.694	+ 35
160	22 58.25	3	0.722	0.693	+ 29
161	23 16.28	4	0.685	0.692	− 7
162	30.50	4	0.681	0.691	− 10
163	1 23 56.44	3	0.686	0.690	− 4
164	2 0 26.13	2	0.623	0.687	− 64
165	1 55.63	2	0.598	0.679	− 81
166	5 39.70	3	0.741	0.665	+ 76
167	6 20.60	4	0.652	0.664	− 12
168	35.20	4	0.686	0.663	+ 23
169	6 47.20	4	0.681	0.662	+ 19
170	7 1.50	4	0.696	0.662	+ 34
171	15.00	5	0.702	0.661	+ 41
172	27.00	4	0.705	0.661	+ 44
173	36.30	5	0.660	0.661	− 1
174	45.60	4	0.733	0.661	+ 72
175	7 59.90	4	0.689	0.661	+ 28
176	8 12.89	4	0.696	0.660	+ 36
177	8 37.08	2	0.694	0.660	+ 34
178	9 15.78	3	0.689	0.660	+ 29
179	32.30	4	0.711	0.660	+ 51
180	+2 9 52.10	4	+0.692	+0.660	+0.032

TABLE I—*Continued*

Normal Number	Phase	Weight	v−b		O−C
			O	C	
181	+2d 10h 11m 30	4	+0M679	+0M660	+0M019
182	26.50	4	0.689	0.660	+ 29
183	36.42	4	0.693	0.660	+ 33
184	45.17	5	0.656	0.660	+ 4
185	50.90	5	0.672	0.660	+ 12
186	10 58.60	5	0.645	0.661	− 16
187	11 6.30	5	0.686	0.661	+ 25
188	13.20	5	0.672	0.661	+ 11
189	20.94	4	0.653	0.662	− 9
190	41.30	4	0.641	0.663	− 22
191	11 54.90	4	0.667	0.664	+ 3
192	12 8.33	3	0.633	0.665	− 32
193	12 37.10	4	0.624	0.667	− 43
194	13 11.30	3	0.637	0.668	− 31
195	30.90	4	0.617	0.669	− 52
196	13 49.10	4	0.634	0.670	− 36
197	14 8.81	2	0.653	0.670	− 17
198	14 36.90	3	0.681	0.672	+ 9
199	15 19.70	3	0.692	0.674	+ 18
200	15 50.90	3	0.666	0.678	− 12
201	16 9.10	4	0.675	0.680	− 5
202	29.40	4	0.630	0.682	− 52
203	45.50	4	0.673	0.687	− 11
204	16 59.20	4	0.633	0.686	− 53
205	17 22.00	3	0.650	0.688	− 38
206	20 1.70	4	0.699	0.699	− 5
207	20 36.56	4	0.739	0.739	+ 31
208	21 20.70	4	0.709	0.713	− 4
209	21 56.50	4	0.699	0.718	− 19
210	+2 22 37.64	2	+0.694	+0.722	−0.028

The influences of ellipticity of figure and of reflection are apparent in the curve. Values of these constants, as well as for the intensity at quadrature, and the depth and the phase of secondary minimum were found by a least-squares solution. Assuming the brightness of the comparison star as 9M94, we have the following magnitudes of Y Camelopardalis: at maximum brightness 10M60; at primary minimum, 12M41; at secondary minimum, 10M68.

The solution of the curve on assumptions of uniform illumination and total darkening at the limb were carried through on the usual lines. The difference in luminosity of the two stars according

to the uniform solution is large. It was impossible to make a satisfactory darkened solution with the retention of all the values found from the least-squares solution. In order to retain the value 0.041 for the difference in brightness of the two sides of the fainter star, it was necessary to increase the depth of secondary minimum by 0.0065, thereby reducing the luminosity of the dark side to zero.

It is interesting to note also that we cannot tell which component is the larger until we know to what degree the disks are darkened

TABLE II

		Uniform	Darkened
Maximum radius of dark star	a_f	0.2451	0.2308
Minimum radius of dark star	b_f	0.2287	0.2152
Maximum radius of bright star	a_b	0.2228	0.2664
Minimum radius of bright star	b_b	0.2079	0.2488
Ratio of radii of each star	b/a	0.934	0.934
Ratio of radii of two stars	k	1.100	0.866
Inclination of orbit plane	i	84° 38′	86° 7′
Component of orbital eccentricity	$e \cos \omega$	+0.018	+0.018
Difference of light of sides of dark star	$2c$	0.041	0.041
Light of bright star	L_b	0.9498	0.9500
Light of brighter side of dark star	L_f	0.0502	0.0410
Light of darker side of dark star	$L_f - 2c$	0.0092	0
Ratio of surface brightness:			
Of the bright star to brighter side of dark star	J_b/J_f	23	17.5
Of the bright star to darker side of dark star		125	∞
Equal-mass densities:			
Of the bright star		0.0418	0.0500
Of the dark star		0.0556	0.0325

at the limb. If the stars present uniformly illuminated disks, then the darker star is slightly the larger. The darkened solution, on the other hand, makes the brighter star considerably the larger.

The curve drawn out in the diagram and given also under "C" in the Table of Normals is the one computed on the assumption of total darkening.

The visual and photographic ranges in primary minimum are practically identical. As the dark star supplies but a small fraction of the light remaining at primary minimum, no conclusions in regard to difference in spectral type of the components can be drawn from this fact.

Other results of interest are summarized in Table II. The unit of length is the radius of the relative orbit. The unit of light is the combined light of the brighter sides of the two stars at maximum area. The mass of each component is assumed equal to that of the sun and the unit of density is the density of the sun.

PRINCETON UNIVERSITY OBSERVATORY
June 1921

THE SPECTROSCOPIC BINARY BOSS 3644, VIRGINIS[1]

By JOHN C. DUNCAN

ABSTRACT

Spectroscopic binary Boss 3644, Virginis.—The elements of the orbit of this sixth-magnitude star of type F2 have been determined from eighteen spectrograms. P is 2.6960 days, K is 24.3 km/sec., γ is 17.6 km/sec., and e is 0.00. A velocity-curve is given.

The radial velocity of the star Boss 3644 ($\alpha = 14^h8^m.5$, $\delta = -22°25'$; 1900.0) was found to be variable by Miss A. M. Brayton from her measures of three spectrograms made with the 60-inch reflector and single-prism spectrograph early in 1920. During the current

TABLE I

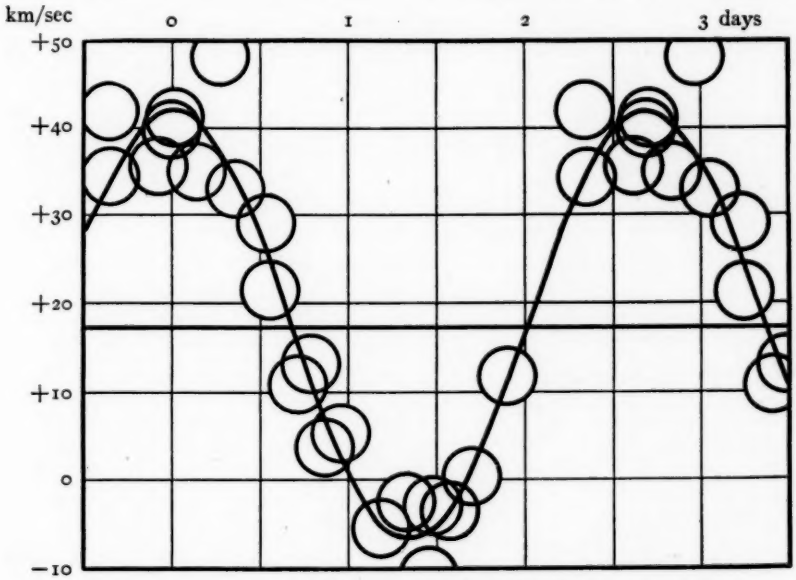
OBSERVATIONS OF BOSS 3644

Plate	Date, G.M.T.	J.D. 2422000+	Days after Maximum Velocity	Observed Velocity	O−C
8990.....	1920 Feb. 5 1^h 06^m	360.046	1.471	− 3.2	+ 2.6
9092.....	Mar. 31 21 57	415.915	0.724	+10.6	− 4.2
9239.....	May 31 20 20	476.847	2.344	+34.2	0.0
9881.....	1921 Jan. 29 1 02	719.043	1.900	+11.7	+ 0.9
9903.....	Feb. 17 23 18	738.971	0.260	+48.0	+10.4
9912.....	Feb. 19 1 14	740.051	1.340	− 2.7	+ 4.0
9925.....	Feb. 20 1 07	741.047	2.336	+41.8	+ 7.9
9930.....	Feb. 20 22 23	741.933	0.526	+28.8	+ 3.0
9943.....	Feb. 23 23 11	744.966	0.863	+ 3.6	− 3.7
9959.....	Feb. 25 22 31	746.938	0.139	+34.9	− 5.8
9972.....	Feb. 26 23 35	747.983	1.184	− 5.6	− 0.6
10017.....	Mar. 19 21 55	768.913	0.546	+21.4	− 3.3
10025.....	Mar. 20 19 38	769.818	1.451	−11.0	− 5.0
10027.....	Mar. 20 22 09	769.923	1.556	− 3.8	+ 0.1
10030.....	Mar. 22 20 17	771.845	0.782	+13.2	+ 1.7
10034.....	Mar. 23 0 30	772.014	0.951	+ 5.2	+ 2.2
10062.....	Mar. 27 19 16	776.803	0.348	+32.8	− 1.6
10096.....	Apr. 17 22 43	797.947	2.620	+35.5	− 6.1
10134.....	Apr. 27 19 31	807.813	1.702	+ 0.2	− 0.9
10142.....	Apr. 28 19 12	808.800	2.689	+39.9	− 2.0
10143.....	Apr. 28 19 36	808.817	0.010	+41.0	− 0.9

[1] *Contributions from the Mount Wilson Observatory*, No. 212.

year, eighteen additional plates have been secured by various observers with the same equipment. All the plates are listed in Table I, in which the column headed "Observed Velocity" gives for each plate the mean result of Miss Brayton's measures and mine. The numbers in the fourth and last columns are derived from the circular elements given below.

The star is of the sixth magnitude and of spectral type F2. From eight to twelve lines were measured on each plate, those



Velocity-curve of Boss 3644

chosen being of iron and occasionally of hydrogen. Preliminary elements were derived by the method of Lehmann-Filhés as follows:

$$P = 2.6960 \text{ days}$$
$$K = 25.0 \text{ km/sec.}$$
$$\omega = 320°$$
$$e = 0.08$$
$$T = \text{J.D. } 2422746.45$$
$$\gamma = +17.47 \text{ km/sec.}$$

A least-squares solution was based on these elements, using only the eighteen plates of 1921 and omitting the period. As the

resulting correction to the eccentricity was -0.11 ± 0.06, this element was assumed to be 0.00 and a second solution was made with T, K, and γ as the only variables. By the first solution Σvv was reduced from 443 to 338, and by the second it was further diminished to 333. The circular elements from the second solution and the period originally found are:

$$P = 2.6960 \text{ days}$$
$$K = 24.34 \pm 1.08 \text{ km/sec.}$$
$$e = 0.00$$
$$T_\Omega = \text{J.D. } 2422744.103 \pm 0.023$$
$$\gamma = +17.60 \pm 0.85 \text{ km/sec.}$$
$$a \sin i = 902{,}300 \text{ km}$$
$$\frac{m_1^3 \sin^3 i}{(m+m_1)^2} = 0.00404 \odot$$

T_Ω indicates the epoch of maximum positive velocity. In the accompanying velocity-curve the radius of the circles representing the observations is equal to the probable error of a single plate, ± 3.18 km/sec.

MOUNT WILSON OBSERVATORY
June 1921